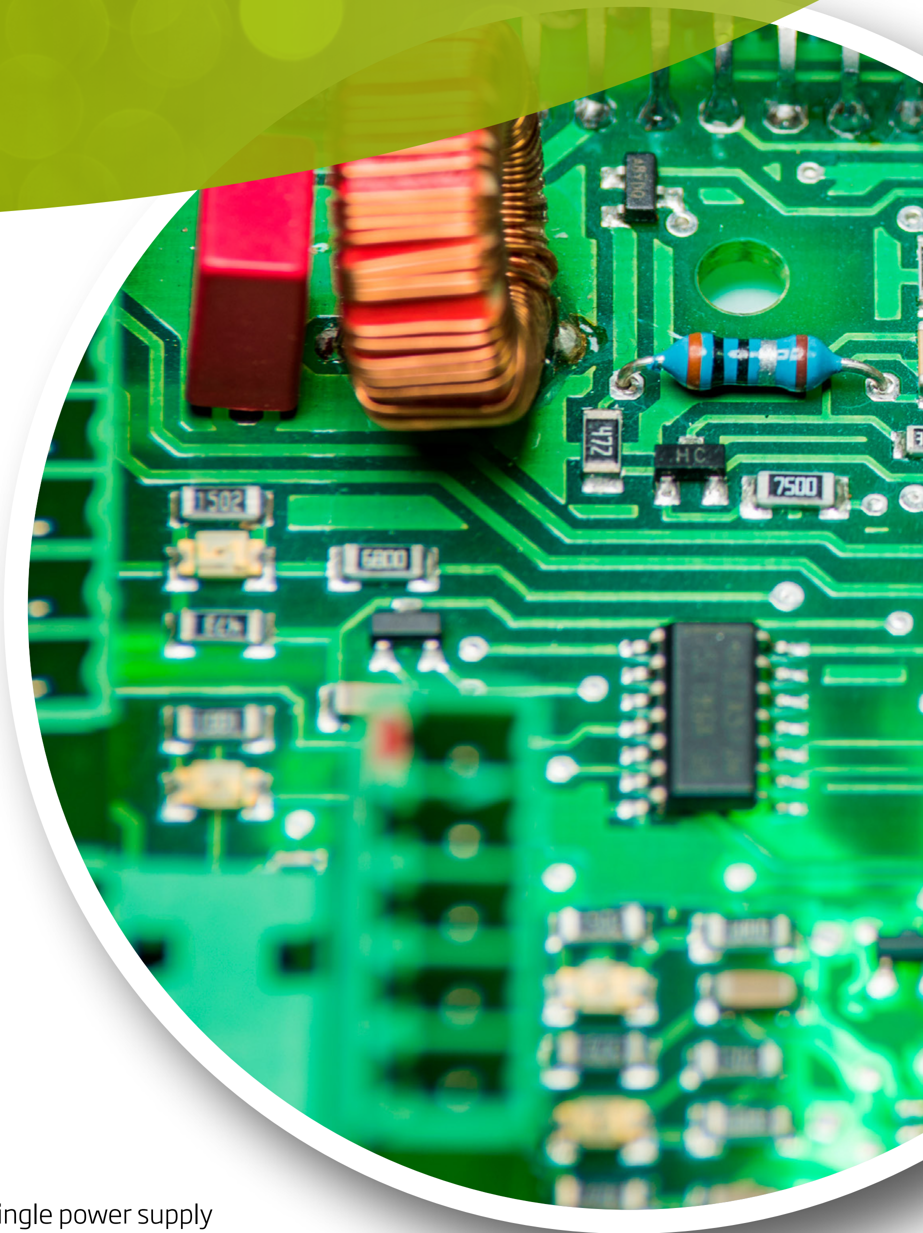
Kirchhoff's laws



Topic areas

Electrical and electronic engineering:

- Ohm's law
- Electrical power
- Kirchhoff's current law (Kirchhoff's first law)
- Kirchhoff's voltage law (Kirchhoff's second law)
- Resistors in series and parallel

Mathematics:

- Simultaneous equations

Prerequisites

None.

Problem statement

Simple circuits, for example those with a single power supply connected in series to a load, are straightforward to analyse in terms of current, voltage drop across loads and power dissipated, using the expressions $V = IR$ and $P = IV$. In practice however, circuits are generally much more complex than this, with multiple components and multiple paths for the current to flow.

How can complex circuits be analysed? What can such an analysis tell us about how the circuit will behave if one or more components fails?

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Activity 1 – Discussion

An engineer has been given the circuit diagram shown in **Figure 1** as a proposed design to power two different designs of LED strip lights and two identical LED bulbs. Discuss how current will flow in the circuit and what requirements and limits must be satisfied to ensure that the components will operate within their design parameters.

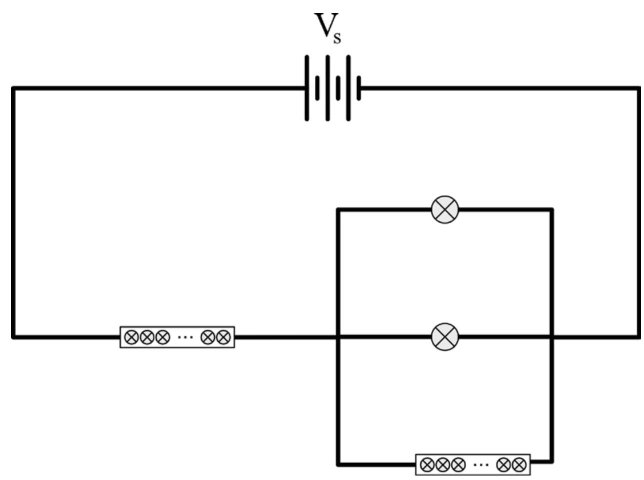


Figure 1
An example circuit

Background – Kirchhoff's first law

Circuit network analysis can be carried out using Kirchhoff's laws. Kirchhoff's first law applies to currents at a junction in a circuit. It states that at a junction in an electrical circuit, the sum of currents flowing into the junction is equal to the sum of currents flowing out of the junction.

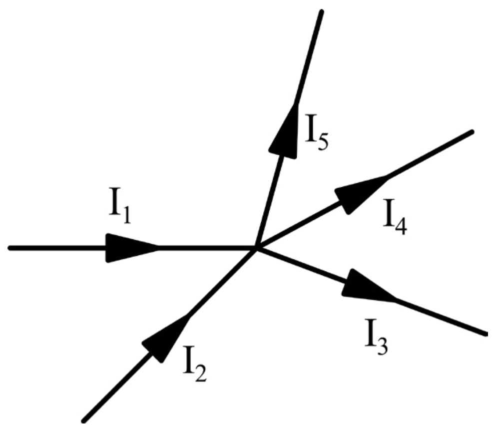


Figure 2
Currents of known direction of flow into and out of a junction

In **Figure 2** the directions of the currents are known. Kirchhoff's first law states that:

$$I_1 + I_2 = I_3 + I_4 + I_5$$

In some cases, the direction of current flow is not known and, in this situation, you can arbitrarily assign a direction, as shown in **Figure 3**. In this example, assuming that all the currents are non-zero, at least one of the currents must have a negative value, indicating that the arrow has been drawn in the wrong direction – current cannot flow into the junction from all three directions without current flowing out.

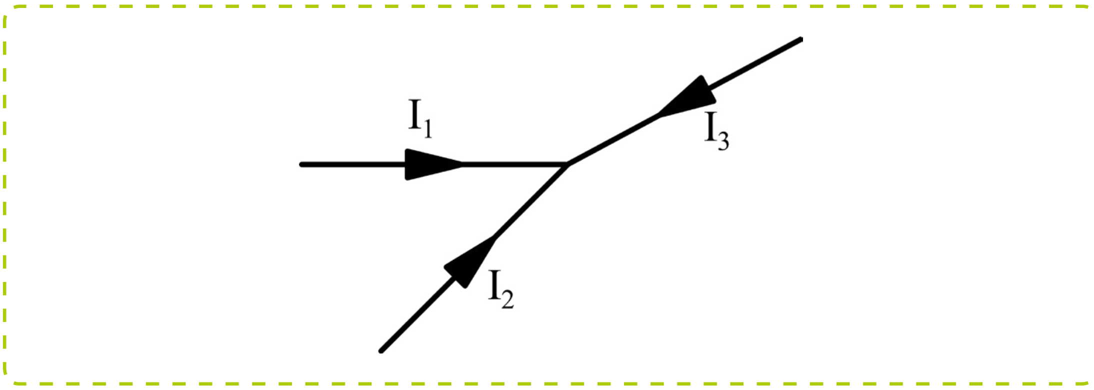


Figure 3
Assumed current direction

Activity 2 – Interactive

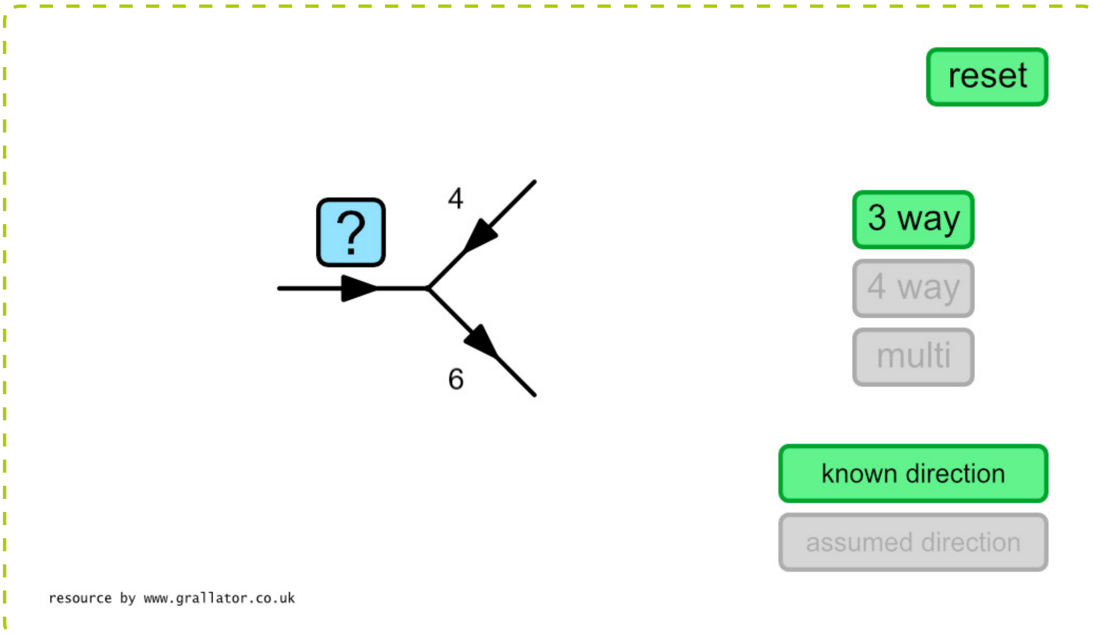


Figure 4
Screen shot of resource

The resource **kirchhoff1** shows a junction in a circuit and lets you practise applying Kirchhoff's first law for various junction configurations. It also allows you to apply the law when the direction of current flow is assumed rather than known. The resource presents an unknown current; use Kirchhoff's first law to determine its value. To reveal the value, click on the unknown box with "?". Click the "reset" box to present another problem. You can change the number of wires by selecting one of the "3 way", "4 way" or "multi" options. You can also test understanding of what happens when you assume a current direction rather than know it – a current value may be negative, indicating that the assumed flow is in the wrong direction.

Background - Kirchhoff's second law

Kirchhoff's second law applies to voltage drops across components in a circuit. It states that around any closed loop in a circuit, the directed sum of potential differences across components is zero. The meaning of 'directed sum' in this definition is explained by **Figure 5**.

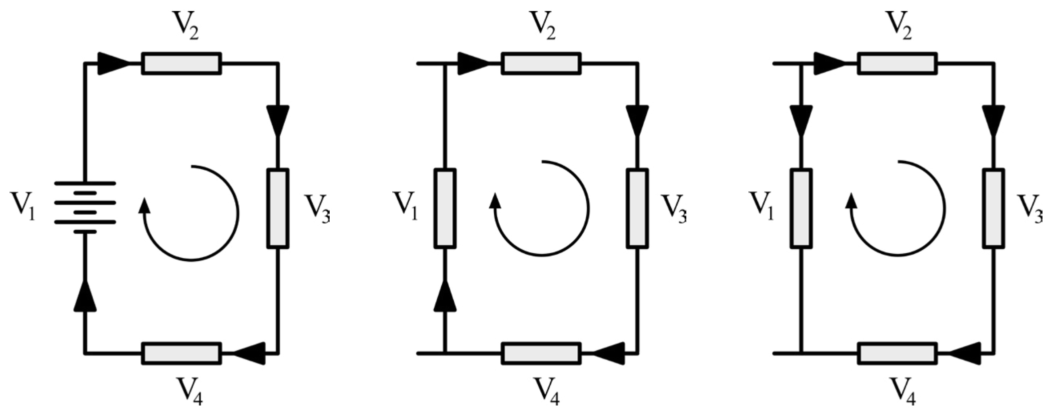


Figure 5a-c

Two closed loops with a power supply (**5a**), without a power supply assumed current direction (**5b**) and without a power supply with actual current direction (**5c**).

Remember:

- For cells, the assumed positive is where the current flows out of the cell.
- For components, the assumed positive is where it flows into the component.

In **Figure 5a**, Kirchhoff's second law states:

$$V_1 - V_2 - V_3 - V_4 = 0$$

The same principle applies in **Figure 5b**, where an assumed current flow has been added. In this case, the diagram has added wires to indicate that it is joined to a larger circuit. This helps clarify that in this case, as there is no dc supply, current must enter and exit the loop.

$$V_1 + V_2 + V_3 + V_4 = 0$$

Note, for the above to be true either all the values must be zero (which means your circuit isn't turned on, or is made of exotic superconductors!), or at least one of the values must be negative, for example the current flows in the opposite direction to that shown by arrow in that (or those) components.

For the case where $V_2 = V_3 = V_4 = 2$ V, V_1 must be −6 V, indicating that the current flowing through the resistor associated with V_1 is as shown in **Figure 5c**, rather than **5b**.

Activity 3 – Interactive

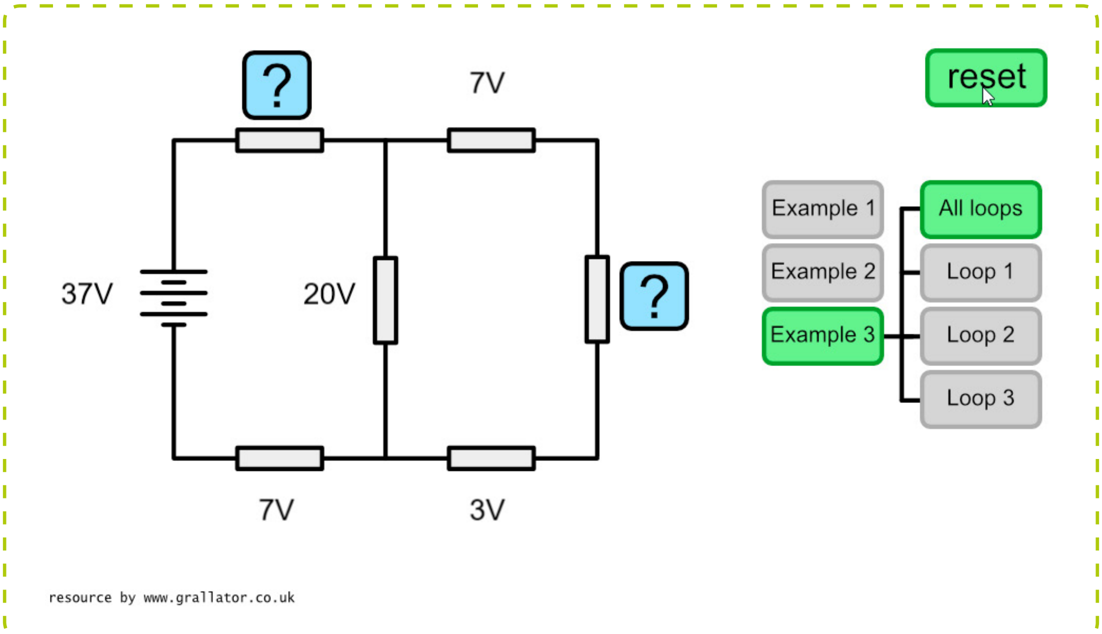


Figure 6
Screen shot of resource

The resource **kirchhoff2** shows either a simple loop with a power source, a simple loop without a power source that is part of a larger circuit, or a more complex circuit. In the complex circuit, the individual loops that can be constructed can be displayed for clarity (note, when all loops are shown, the assumed current flow is not shown). This resource works in a similar way to the previous resource. It presents one or two unknown voltages, and you can use Kirchhoff's second law to determine the value or values. To reveal the value, click on the unknown box with "?". Click the "reset" box to present another problem.

Activity 4 – Analysing the lighting circuit using Kirchhoff's laws

Look again at the suggested lighting circuit.

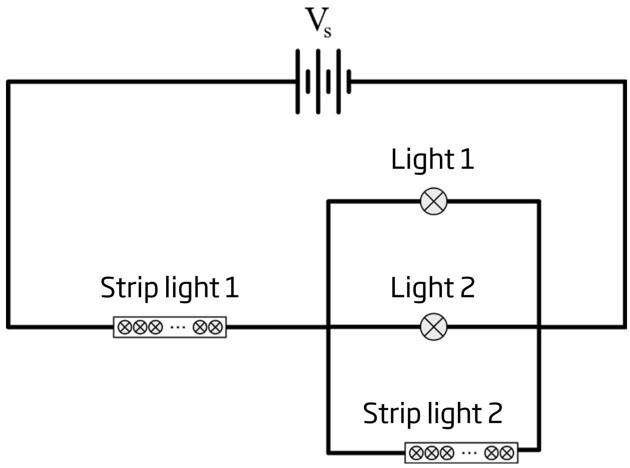


Figure 7
An example circuit

1) Draw the expected current flows in each part of the circuit.

The resource **kirchhoff3** can be used to show the current flows and help identify the loops and junctions in the circuit.

The following table gives the voltage drop characteristics of the strip lights and lights used in the circuit.

	Strip light 1	Strip light 2	Light 1	Light 2
Operating voltage (V)	24 V	12 V	12 V	12 V

2) Why do strip light 2, light 1 and light 2 all have the same operating voltage?

3) Identify a suitable closed loop and use Kirchhoff's second law to determine the voltage of the power supply.

4) A current of 600 mA is drawn from the power supply. Calculate the resistance of and power consumed by strip light 1.

5) Strip light 2 consumes 3.6 W of power. Calculate the current flowing through strip light 2 and its resistance.

6) Light 1 is identical to light 2. Use Ohm's Law to show that they have the same current passing through them. Use Kirchhoff's first law and the result from (5) to calculate this current.

7) Calculate the resistance of light 1 and light 2, and the power each consumes.

8) Explain why the circuit is not a good design, thinking in particular about what would happen if one of the components fails.

Stretch and challenge activity

Calculate the voltage drops across and the currents passing through the components, and the power consumed by each component in the circuit under the conditions that strip light 2 fails and no current can pass through it. Fill in the table of values and compare the values with those calculated when all the components are working.

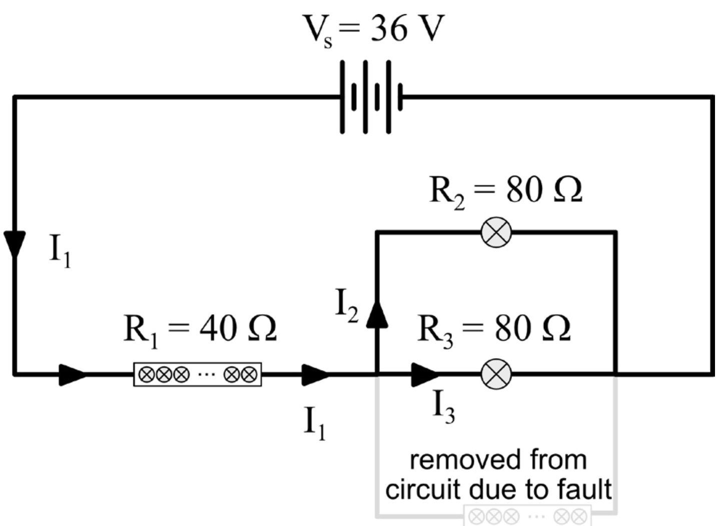
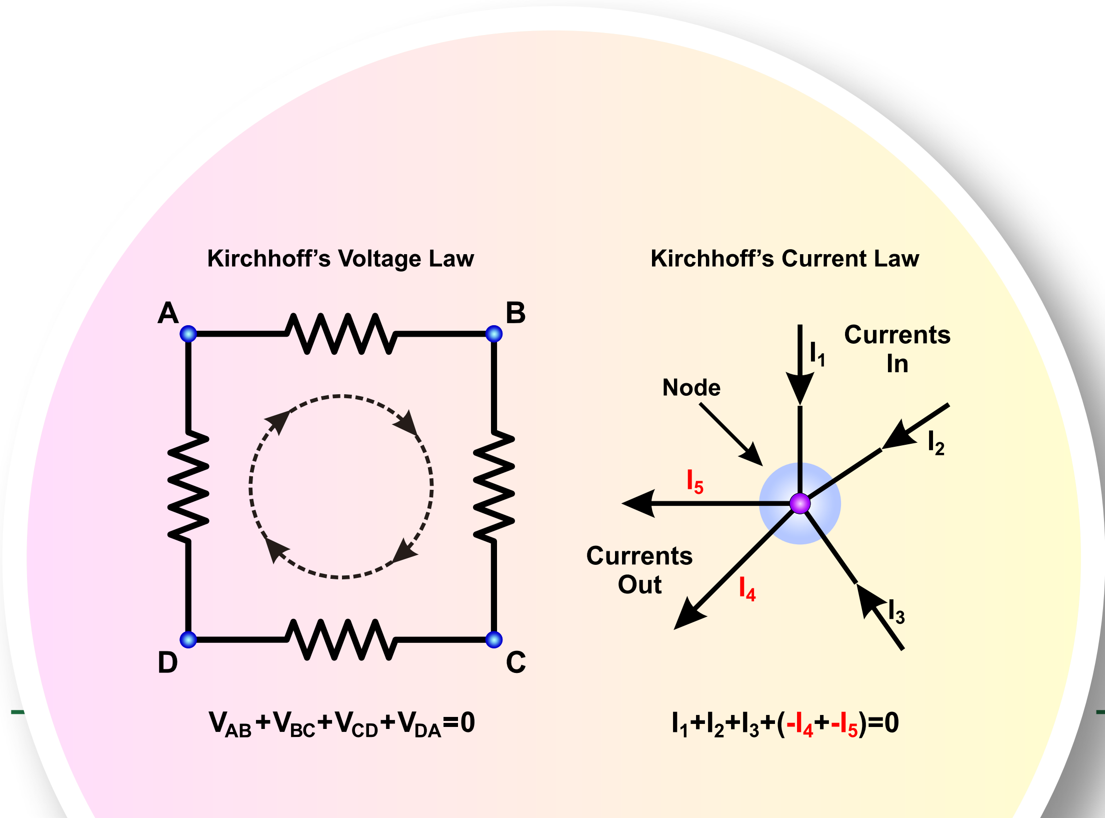


Figure 8
Lighting circuit with a fault

	Voltage drop across component		Current through component		Power consumed by component	
	No fault	Fault	No fault	Fault	No fault	Fault
Strip light 1	24 V	____V	600 mA	____ mA	14.4 W	____W
Light 1	12 V	____V	150 mA	____mA	1.8 W	____W
Light 2	12 V	____V	150 mA	____mA	1.8 W	____W



Notes and solutions

Activity 1

The current will flow from the source in an anti-clockwise manner through the circuit. Where the circuit branches into three parallel paths, the current will divide, flow through the individual components and combine again where the paths re-join.

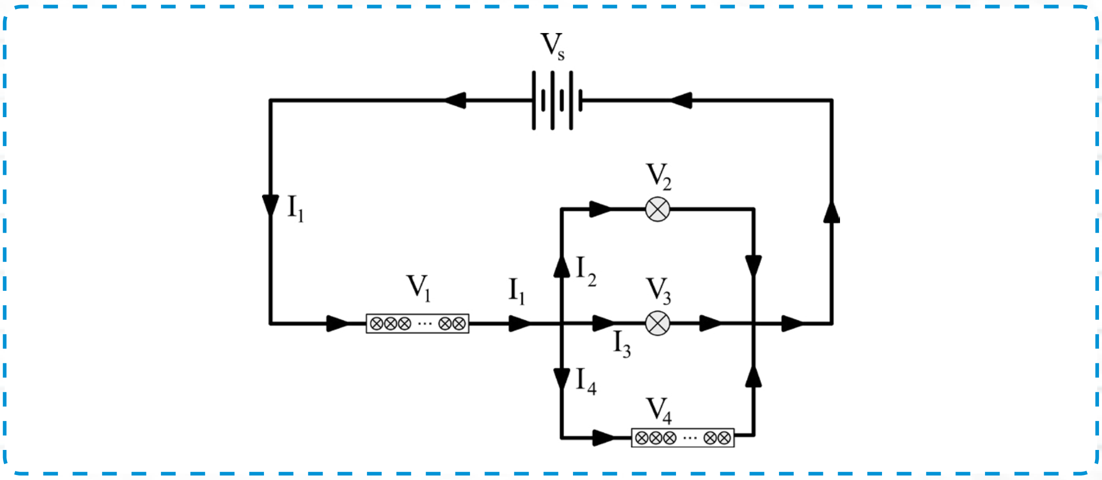


Figure 9

- The power source should have the correct supply voltage be able to supply enough current to power the whole circuit.
- The current through and potential difference across the individual components should be enough to power them satisfactorily.
- The current through and potential difference across the individual components must not be so high that they are damaged.

Activity 4

1) The resource kirchhoff3 shows the expected current flows. It also allows you to view the two junctions and the six closed loops that can be formed by pressing the green arrow keys to cycle through them.

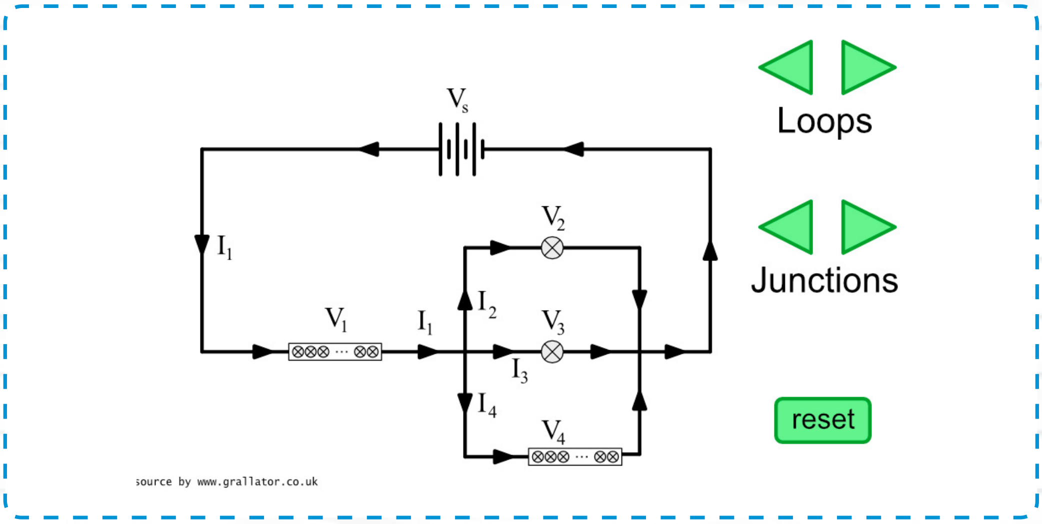


Figure 10 Screenshot of resource

2) The voltages across light 1 and light 2 must be the same as they are connected in parallel.

3) There are three possible loops that can be identified – any of these will let you determine the voltage of the power supply.

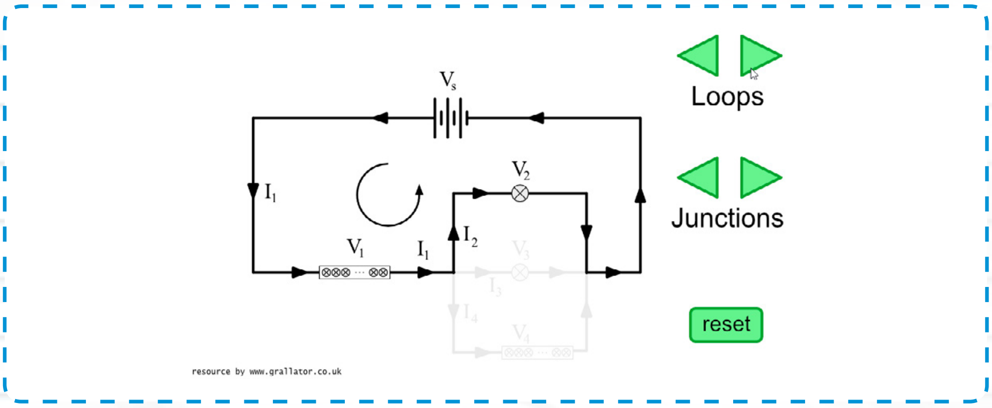


Figure 11

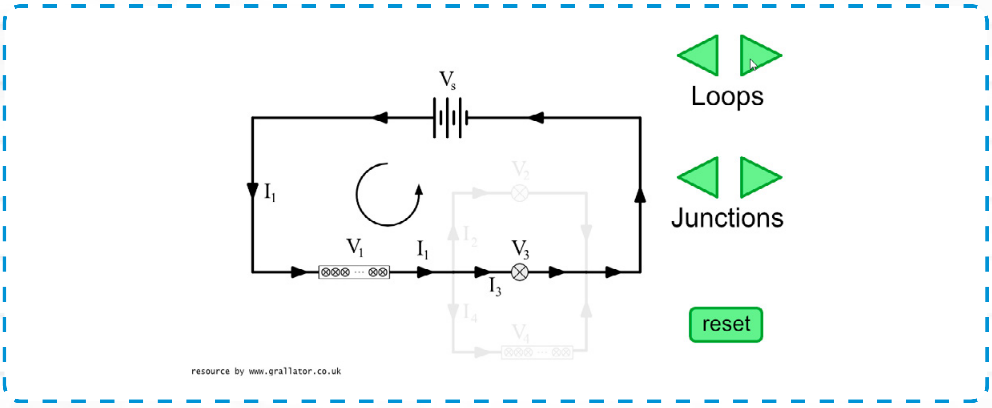


Figure 12

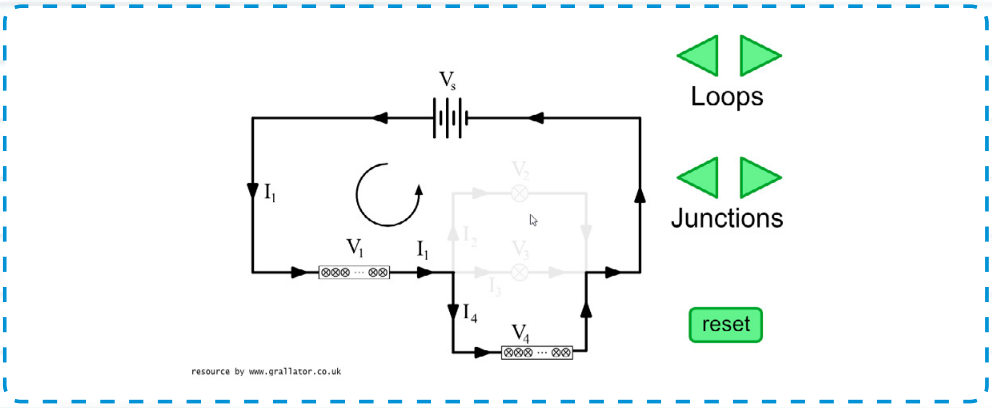


Figure 13

The voltage drop V_1 is 24 V, the voltage drops V_2, V_3 and V_4 are all 12 V. Kirchhoff's second law gives:

$$V_s = 24 + 12 = 36\ V$$

4 The voltage dropped across strip light 1 is 24 V and a current of 600 mA (=0.6 A) flows through it. The resistance is given by:

$$V = IR \Rightarrow R = \frac{V}{I} = \frac{24}{0.6} = 40\ \Omega$$

The power is given by:

$$P = IV = 0.6 \times 24 = 14.4\ W$$

5 Strip light 2 consumes 3.6 W of power and has a voltage drop across it of 12 V. The current flowing through it is given by:

$$P = IV \Rightarrow I = \frac{P}{V} = \frac{3.6}{12} = 0.3\ A = 300\ mA$$

The resistance of light strip 2 is given by:

$$V = IR \Rightarrow R = \frac{V}{I} = \frac{12}{0.3} = 40\ \Omega$$

6 Applying Kirchhoff's second law to a closed loop including lights 1 and 2.

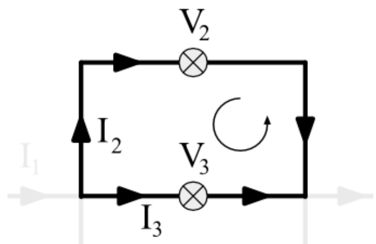


Figure 14

As these two lights are in parallel, $V_3 - V_2 = 0 \Rightarrow V_3 = V_2$, which agrees with the stated operating voltages. For light 1, the relationship between the voltage, current and resistance is given by Ohm's Law as $V_2 = I_2R_2$. Similarly, for light 2, $V_3 = I_3R_3$.

As the two bulbs are identical they must have the same resistance, $R_2 = R_3 = R$ and as $V_3 = V_2$,

$$I_2R_2 = I_3R_3$$
$$I_2R = I_3R$$

$$I_2 = I_3$$

Looking at the following junction:

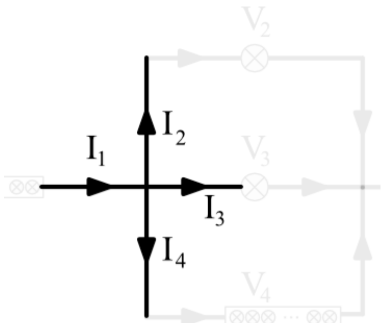


Figure 15

Kirchhoff's first law states that $I_1 = I_2 + I_3 + I_4$.

Previous calculations have shown that I_1 = 600 mA, I_4 = 300 mA and that $I_2 = I_3$. Using these values:

$$600 = I_2 + I_3 + 300$$
$$300 = 2I_2$$
$$I_2 = 150\ mA$$

This implies $I_3 = 150\ mA$.

7 Using Ohm's Law:

$$V = IR \Rightarrow R = \frac{V}{I} = \frac{12}{0.15} = 80\ \Omega$$

The power consumed is:

$$P = IV = 0.15 \times 12 = 1.8\ W$$

8 The circuit is not a good design as it is not fault tolerant. If light strip 1 failed then no current would flow and none of the other lights would work. If lights 1 and 2, or light strip 2 failed, the other lights, including light strip 1, would still illuminate. However, there would be a change in the overall resistance of the circuit that would change the voltage dropped over the components and currents flowing through them. This might lead to a reduction in brightness, or a damaging increase.

Stretch and challenge activity

In this problem it is known that $R_1 = 40\ \Omega$, $R_2 = R_3 = 80\ \Omega$.

Answer seven in the previous activity is still valid so that $V_2 = V_3$, $R_2 = R_3$ and $I_2 = I_3$ as before.

Kirchhoff's first law applied to the junction gives $I_1 = I_2 + I_3$, but as $I_2 = I_3$, this implies $I_1 = 2I_2 = 2I_3$.

Using Kirchhoff's second law can be applied on the following loop.

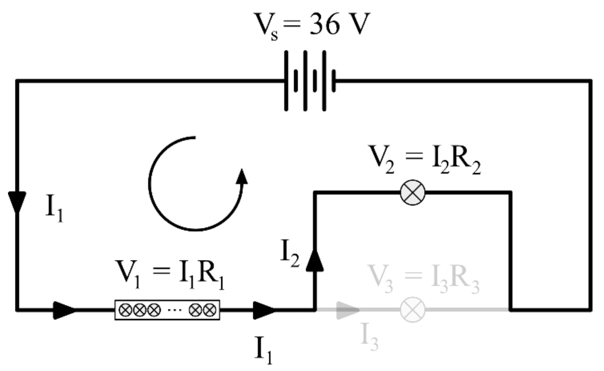


Figure 16

Noting that the voltage drop over a component is, by Ohm's Law, the product of the current and resistance:

$$36 = V_1 + V_2$$
$$36 = I_1R_1 + I_2R_2$$
$$36 = 40I_1 + 80I_2$$

Substitute $I_1 = 2I_2$

$$36 = 80I_2 + 80I_2$$
$$36 = 160I_2$$
$$I_2 = 0.225\ A$$

So that $I_1 = 0.45\ A$ and $I_3 = 0.225\ A$.

Using these results:

$$V_1 = I_1R_1 = 0.45 \times 40 = 18\ V$$
$$V_2 = V_3 = I_2R_2 = 0.255 \times 80 = 18\ V$$

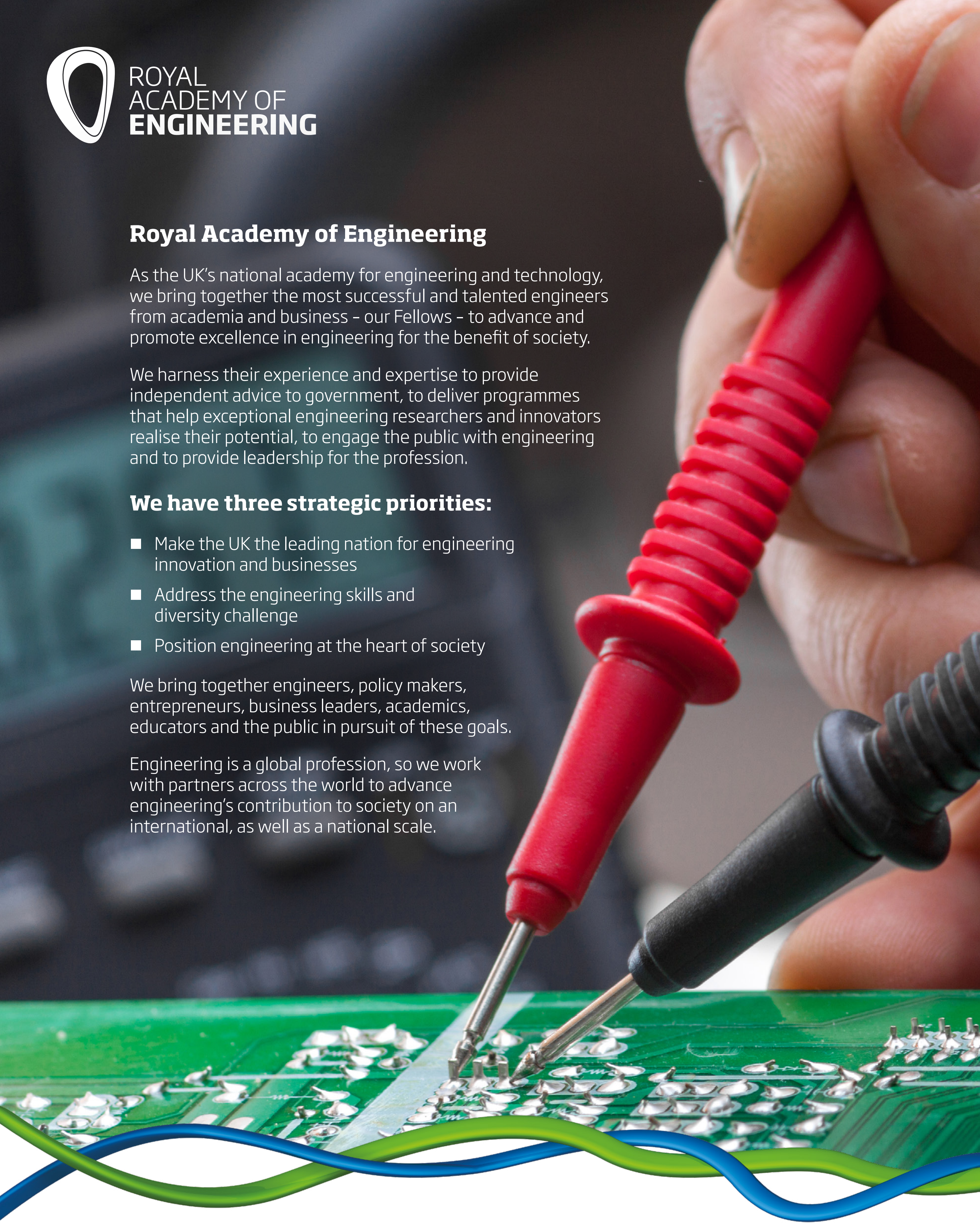
The power consumed by each component is given through $P = IV$ as:

$$P_1 = I_1V_1 = 0.45 \times 18 = 8.1\ W$$
$$P_2 = I_2V_2 = 0.225 \times 18 = 4.05\ W$$
$$P_3 = I_3V_3 = 0.335 \times 18 = 4.05\ W$$

The values are compared in the table below.

	Voltage drop across component		Current through component		Power consumed by component	
	No fault	Fault	No fault	Fault	No fault	Fault
Strip light 1	24 V	18 V	600 mA	450 mA	14.4 W	8.1 W
Light 1	12 V	18 V	150 mA	225 mA	1.8 W	4.05 W
Light 2	12 V	18 V	150 mA	225 mA	1.8 W	4.05 W

Under fault conditions the strip light has a lower voltage drop, current and power, which will dim the light. The other two lights will see an increase in voltage, current and power consumed. These lights will be extra bright and may be damaged when operating under these conditions.



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