Friction and sliding

Topic areas

Mechanical engineering:

- Forces
- Friction

Mathematics:

- Trigonometric calculations and identities

Prerequisites

It may be useful to look at the resource **'Forces, centre of gravity, reactions and stability'** to introduce where forces act on a simple object and the conditions for toppling stability.

Problem statement

There are many situations where knowing whether an item will slide on an inclined plane is important either for safety or correct operation.

For example, packages moving around a distribution centre may need to be transported on inclined rubber conveyor belts without sliding, but also be delivered to chutes that they should slide down freely. How can an engineer determine whether an object will slide or remain in place on an inclined plane?

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Background

The weight of an object is the force due to gravity acting between the earth and the object, and is measured in newtons (N). As the scale of the earth is large compared with many everyday objects, the weight force acting on an object, F, can be related to its mass, m, through the approximating expression

$$F = mg,$$

where g is the acceleration due to gravity, usually taken to be 9.81 ms^{-2} at the earth's surface.

When a force acts on a body the body will accelerate in the direction of the force unless there is a balancing force to oppose it. For an object sitting on a plane, this balancing force is the normal reaction, R, which acts normally and upwards (or perpendicular) to the plane. If there is friction between the object and the plane which acts against any sliding tendencies, a frictional force acts upward along the plane.

Activity 1 – Discussion

Look at the stationary blocks, shown in **Figure 1**, on a level plane (left) and an inclined plane (right).

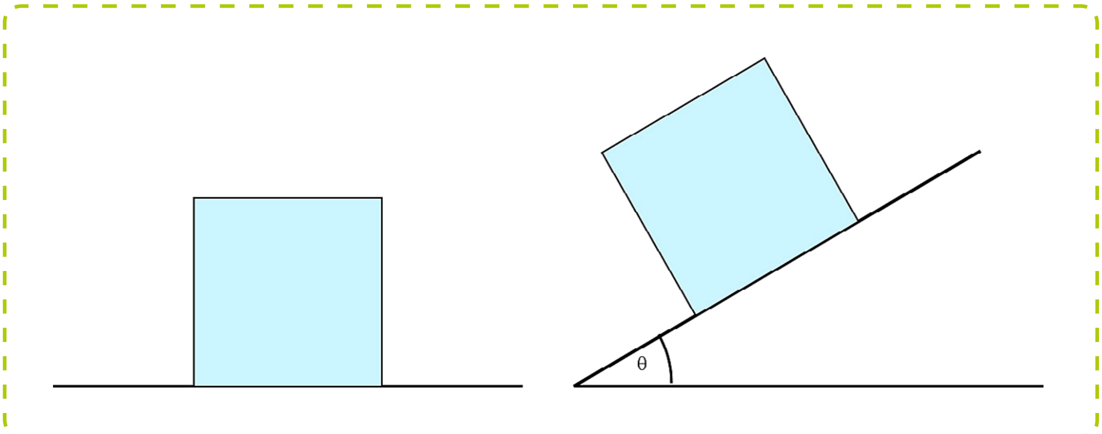

Figure 1 Stationary blocks

Discuss what forces are acting and draw force arrows showing where they are acting. What assumptions do you make?

Background – Friction

In **Figure 1** you should have sketched the weight of the object acting vertically down, a normal reaction from the plane and a friction force parallel with the plane and acting upwards. As the latter two forces will be determined by the weight force, it is useful to resolve the components of the weight force normal to, and parallel with the plane.

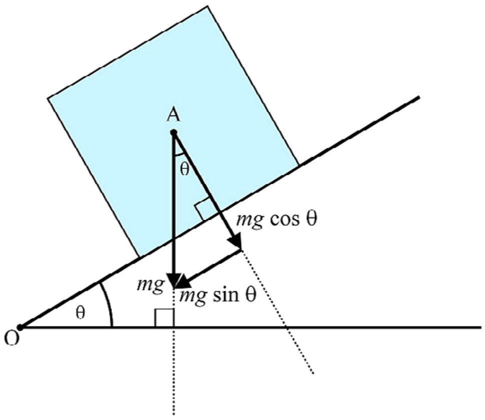

Figure 2

The left-hand side of **Figure 2** shows

Component of weight force normal to the plane: $mg\cos\theta$

Component of weight force down the plane: $mg\sin\theta$

These results are based on the fact that the angle at O equals the angle at A. This can be shown by considering the two right angled triangles on the right-hand side of **Figure 2**. The two marked angles α and β must be equal (as opposite angles on intersecting angles are equal). As these angles are equal, and both triangles also have a right angle, the remaining angles, i.e. the ones at O and A must also be equal, and have a value of θ.

For the object to be static on the inclined plane, the magnitude of the frictional force that resists sliding, F (in newtons), must be equal to the magnitude of the component of the object's weight acting down the plane, and act in the opposite direction. That is, for a mass m (in kg) on a plane inclined at an angle of θ to the horizontal (see **Figure 2**):

$$F = mg\sin\theta$$

Experience shows that the frictional force does not always balance the component of the object's weight acting down the plane. In some cases, when the plane is tilted enough, the object starts to slide down the plane indicating that the friction force is less than the component of the object's weight acting down the plane, i.e. there must be a limit on the force that friction can apply.

A limit on the magnitude of the frictional force, F (in newtons), is modelled by an inequality between it, the normal reaction, R (in newtons), and the friction characteristics between the object and the plane, modelled by a parameter μ (which has no units) as

$$F \leq \mu R$$

where μ is the coefficient of friction.

The interpretation is

- While the tilt angle of the plane is not sufficient for the object to start sliding, the frictional force exactly balances the component of the object's weight acting down the plane, $F = mg \sin\theta$. Note, this also means that $F < \mu R$.
- The point of sliding occurs when the upper limit of the frictional force is reached, i.e. $F = \mu R = mg \sin\theta$.
- The magnitude of F cannot exceed μR. When the component of the object's weight acting down the plane is larger the μR the object slides. In this case $mg \sin\theta > \mu R$ and $F = \mu R$.

Some typical values of μ are given in **Table 1**.

Table 1
Some typical coefficients of friction

Materials	μ
Car tyre on a dry road	1.0
Car tyre on a wet road	0.2
Steel block on ice	0.03
Aluminium block on an aluminium surface	1.2

Notice μ can be greater than 1.

A summary of all the forces and coefficients used in a friction-limited sliding problem is given in **Table 2**.

Table 2

mg	**The weight of the object.**
$mg \cos\theta$	**The component of the weight normal to the plane.**
$mg \sin\theta$	**The component of the weight along the plane**
R	**The normal reaction of the plane. This should equal the component of the weight normal to the plane.**
μ	**The coefficient of friction.**
μR	**The maximum friction force the surface contact can provide**
F	**The actual friction force that applies.**

Activity 2 - Sliding

The resource **sliding-1**, shown in **Figure 3**, allows you to tilt an object with a uniform mass distribution and a mass of 1 kg.

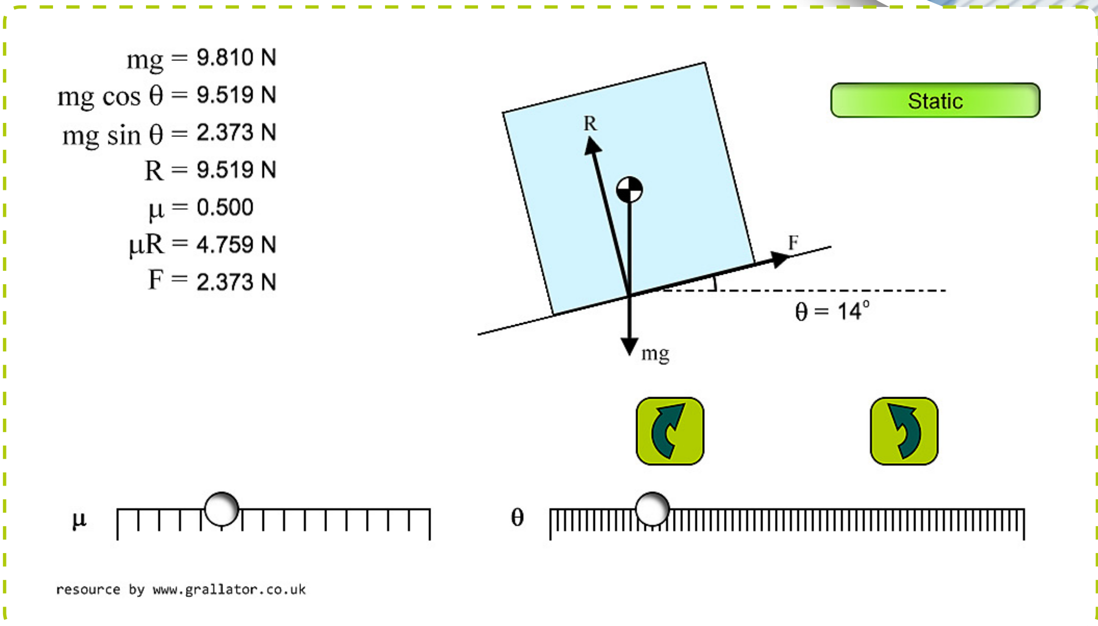

Figure 3
Screen shot of resource

The object can be tilted by dragging the slider on the right and/or by clicking/tapping to buttons for fine control. Additionally, the coefficient of friction between the object and the surface can be changed by dragging the slider on the left.

1. Investigate the angle at which the object becomes unstable and wants to slide. Verify the values and limits for the magnitude of the frictional force are as described in the background section, above, such as:
   - The components of the weight force normal to and down the plane are $mg \cos \theta$ and $mg \sin \theta$ respectively.
   - The normal reaction balances the component of weight normal to the plane.
   - When the object is static, the friction force balances the component of weight down the plane.
   - The friction force is always less than or equal to μR.
   - When the object slides the friction force is limited to a value μR, and this is less than the component of weight down the plane.
2. What do you notice about the point at which the object slides?
3. What happens when the coefficient of friction is large, or when the object is inclined at a very steep angle?

Activity 3 – Finding the sliding angle

The diagram in **Figure 4** shows a uniform block of mass mg on a plane inclined at an angle θ.

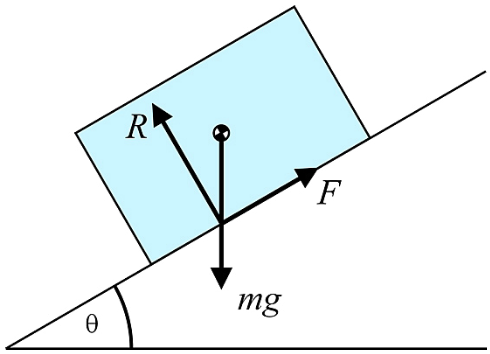

Figure 4
Forces acting on a uniform block

1. Use the information given in the background to calculate the angle at which the object will start to slide.
2. Calculate the sliding angle when μ = 0.7 and compare the results with the predictions of the resource **sliding-1**.

Stretch and challenge activity

It can be shown that an object of uniform density on an inclined plane will tip when the angle of the incline, θ, satisfies

$$\tan\theta = \frac{w}{h}$$

Where w is the object width and h is the object height, see **Figure 5**. (For details see the activity **'Centre of gravity, reactions and stability'**.)

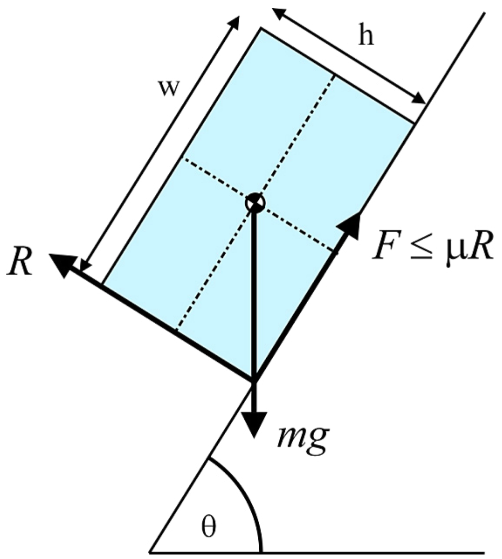

Figure 5
An object on a rough plane at the tipping point

1. Under what conditions will an object of uniform density and with width w and height h, on plane inclined at an angle θ and with a coefficient of friction of μ:
   a) Slide
   b) Tip
   c) Remain stationary
2. What happens after the object has tipped?

Notes and solutions

Activity 1

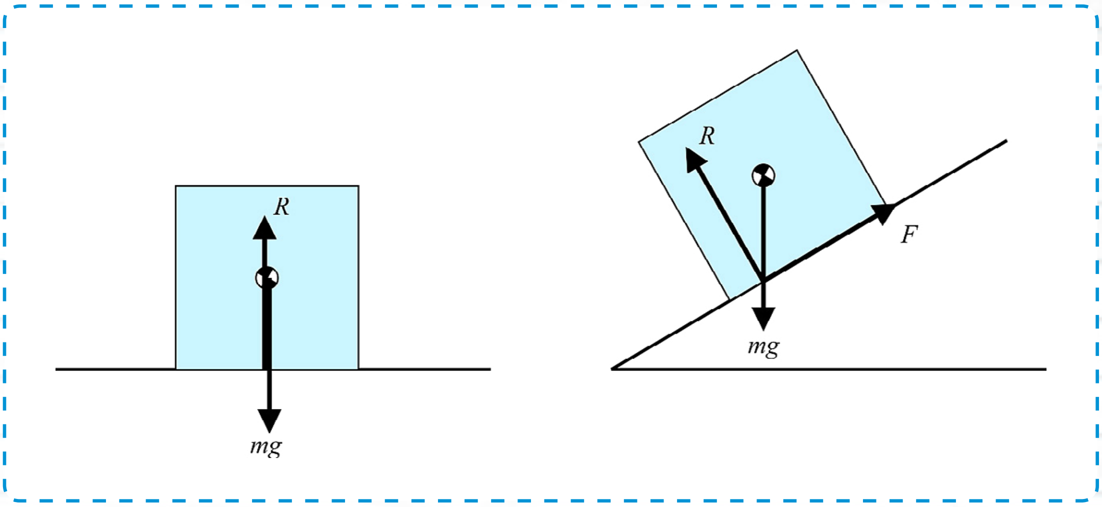

Figure 6
Force diagram

It is stated that the object is stationary, therefore there should be no net forces acting.

On the left-hand diagram, the weight force mg acts vertically downwards through the centre of gravity of the object. This is balanced by an equal and opposite normal reaction force, R. This force acts between the base of the object and the plane and through the centre of gravity.

On the right-hand diagram the weight force mg acts vertically downwards through the centre of gravity of the object. Again there is a normal reaction force, R, acting perpendicular to the plane. It is drawn acting at the base of the object where the line of action of the weight force intersects the base. As the object is stationary, there must also be a friction force acting along the plane to prevent sliding. This is also drawn on the base of the object acting from the same point as the normal reaction.

Activity 2 - Sliding

1. The resource sliding-1 shows a number of values, which are described in **Table 2**.

mg	**The weight of the object.**
$mg \cos \theta$	**The component of the weight normal to the plane.**
$mg \sin \theta$	**The component of the weight along the plane**
R	**The normal reaction of the plane. This should equal the component of the weight normal to the plane.**
μ	**The coefficient of friction.**
μR	**The maximum friction force the surface contact can provide**
F	**The actual friction force that applies.**

Table 2

2. Changing the angle modifies all but mg and μ in the above table. As the angle is increased, the component of weight along the plane, $mg \sin \theta$, increases. To balance this, the frictional force, F, also increases. This continues until F reaches the limit of μR, at which point it no longer balances the component of weight along the plane. In this case there is a net force down the plane and the object starts to slide. Notice that the limiting value of μR also changes with the angle, as R is equal to $mg \cos \theta$.

3. When the coefficient of friction is large it is possible for the angle of inclination to take the line of action of the object's weight out of its base before the conditions for sliding occur. In this case the object topples over rather than slides.

   When the angle is very large, the line of action of the object's weight can be out of its base. Even if the object is sliding, this makes the object unstable as the furthest forward the friction can act is on the lower corner of the object. In this case the object will topple.

Activity 3 - Finding the sliding angle

1 Resolving forces normal to the plane

$$R = mg\cos\theta$$

Resolving forces along the plane, when the object is not sliding:

$$F = mg\sin\theta$$

However, $F \le \mu R$ and the limit of the friction force occurs when $F = \mu R$ (at this point the object is just about to slide). Substituting

$$F = \mu R = mg\sin\theta \Rightarrow \sin\theta = \frac{\mu R}{mg}$$

Substituting for R

$$\sin\theta = \frac{\mu R}{mg} = \frac{\mu mg\cos\theta}{mg} = \mu\cos\theta$$

Rearranging

$$\sin\theta = \mu\cos\theta$$
$$\frac{\sin\theta}{\cos\theta} = \mu$$
$$\tan\theta = \mu$$

i.e. the object slides when the tangent of the angle of the inclined plane is equal to the coefficient of friction.

2 For the case where $\mu = 0.7$:

$$\tan\theta = \mu = 0.7$$
$$\Rightarrow$$
$$\theta = 34.99^{\circ}$$

The resource **sliding-1** can only work with integer angles. You will note however, that the object is static at an angle of 34°, and sliding at an angle of 35°, which agrees with the above prediction.

Stretch and challenge activity

1 From **Activity 3**, the object will slide at an angle θ_s, where $\tan\theta_s = \mu$, and it has been stated that the object will tip over at an angle θ_t, where $\tan\theta_t = \frac{w}{h}$.

a) If $\mu < \frac{w}{h}$ then $\theta_s < \theta_t$, i.e. the sliding angle is less than the tipping angle. In this case the object will slide before tipping over, and sliding will occur when $\theta > \theta_s$. Note, if $\theta > \theta_t$ the object will topple rather than slide. If the object does topple, what happens next depends on the values of w and h, see below.

b) If $\mu > \frac{w}{h}$ then $\theta_s > \theta_t$, i.e. the sliding angle is greater than the tipping angle. In this case the object will tip over before sliding, and the tipping will occur when $\theta > \theta_t$. What happens next depends on the values of w and h, see below.

c) The object will remain stationary when the angle of incline satisfies both $\theta < \theta_s$ and $\theta < \theta_t$, where $\tan\theta_s = \mu$ and $\tan\theta_t = \frac{w}{h}$.

2 After toppling the object will have taken a 90° rotation which will switch the height and width dimensions, resulting in a new topple angle θ_t, as shown in **Figure 7**.

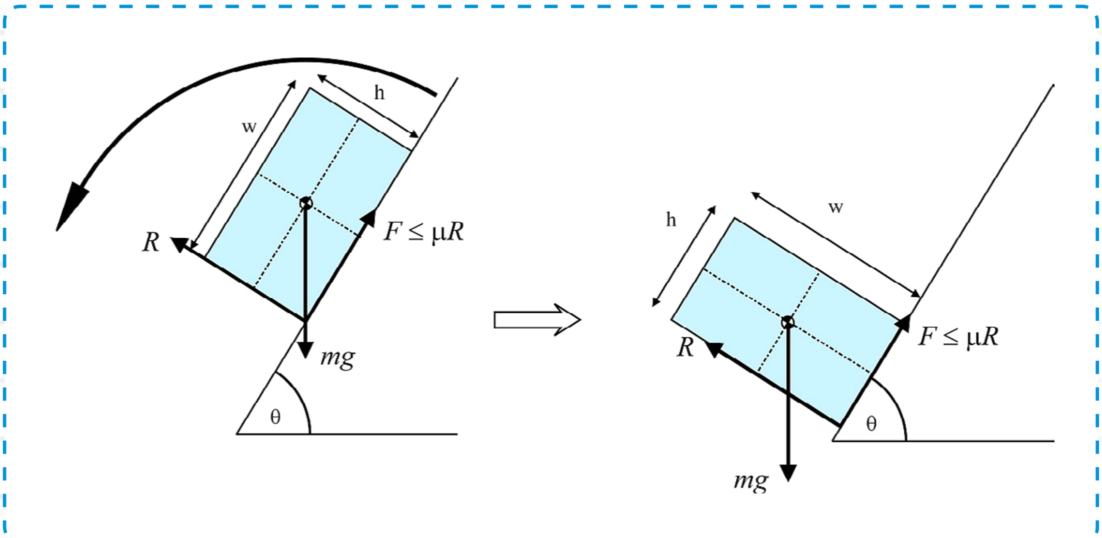

Figure 7

The new configuration may have a value of θ_t that is smaller than the previous one (as exemplified above). In this case the toppling will continue. However, if the new configuration has a larger value of θ_t then the object would not topple a second time. The sliding angle is not affected by toppling, so in this new configuration, and providing $\theta > \theta_s$, the object will transition from toppling to sliding. If $\theta > \theta_s$ is not satisfied, the object will stop.

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