AC power and power factor

Topic areas

Electrical and electronic engineering:

- ✓ AC true, reactive and apparent power
- ✓ AC power factor

Mathematics:

- ✓ Vector addition
- ✓ Pythagoras' theorem
- ✓ Trigonometry

Prerequisites

'AC phasors and fault detection' resource to introduce phasor diagrams and the relationship between resistance, reactance and impedance.

Problem statement

The relationship between voltage and current in an AC system is determined by the impedance of the circuit.

The impedance can consist of a resistive component and a reactive component, the latter of which introduces a phase difference between voltage and current.

How does the phase difference affect the power drawn by the circuit (in this case a motor)?

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Activity 1 - Discussion

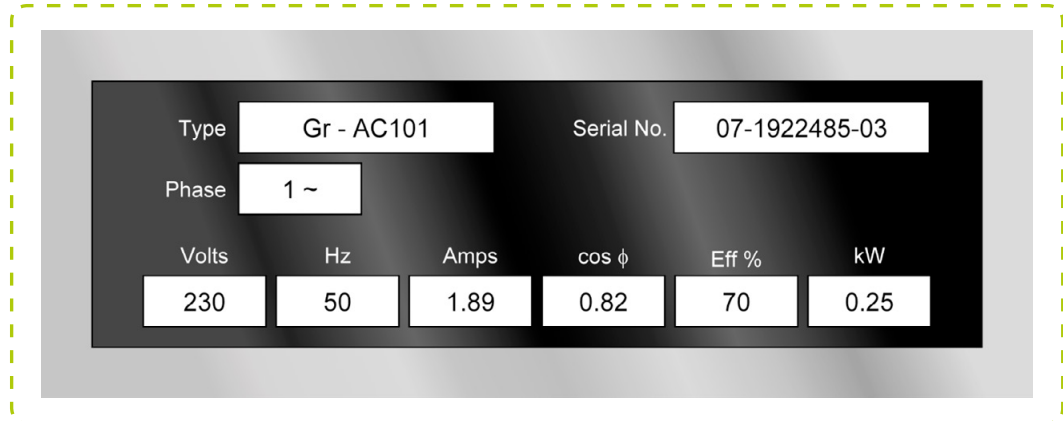**Figure 1** shows an example of a nameplate attached to an AC motor. Discuss what each of the values might represent.

Type	Gr - AC101		Serial No.	07-1922485-03
Phase	1 ~			

Volts	Hz	Amps	cos φ	Eff %	kW
230	50	1.89	0.82	70	0.25

Figure 1
Motor nameplate

Activity 2 - Analysis of the values

The electrical power consumed by the motor, P, is given by the product of the current drawn, I, and the voltage drop, V, i.e. $P = IV$. Calculate

1 The electrical power drawn by the motor.

2 The expected useful power output if the motor turns input power to output power with the efficiency shown on the nameplate.

Discuss the result and investigate if any of the other values given on the nameplate can be relevant, and if so, how, to obtain the power stated for this motor.

Activity 3 - Impedance diagram for the motor

The electrical characteristics of this motor are given in Table 1.

Resistance	100 Ω
Inductance	220 mH

Table 1

1 Calculate the reactance, X, for the motor.

2 Draw an impedance triangle and calculate the impedance and phase angle.

3 Verify that when connected to a 230 V AC supply, the motor will draw the current shown on the nameplate (1.89 A).

The resource **lcr-series-1** can be used to automate the calculation of inductance, impedance, phase angle and draw the phasor diagram.

Activity 4 – Power Phasor

The resistance and inductance in the motor are in series, which means that the same current flows through the resistive and inductive components. Ohm's law for each component of the circuit is:

Resistive component $\quad V_R = IR$

Reactive component $\quad V_X = IX$

Overall circuit $\quad V = IZ$

This can be shown on a voltage phasor diagram as shown in **Figure 2** (using **lcr-series-1**; click the $V = IZ$ option button).
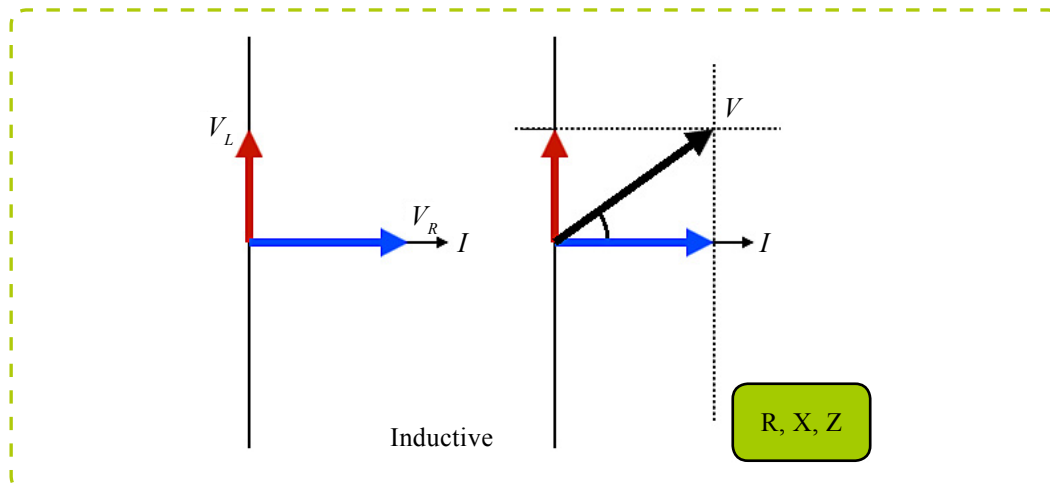

Figure 2 Partial screen of lcr-series-1

For the whole circuit $P = IV$ was used to calculate the electrical total power consumed by the motor, where V and I are related through $V = IZ$.

1 Using this, write down expressions for the resistive and reactive power components of the motor.

2 Use the values on the nameplate shown in **Figure 1**, and the values calculated in Activity 3 to calculate values for the resistive and reactive power components for this motor. Plot the values on a diagram and calculate the vector sum of the resistive and reactive powers. Compare the result with the total electrical power consumed by the motor (see activity 2).

3 Find the cosine of the phase angle and compare it with the cos ϕ stamped on the motor.

4 Activity 2.2 showed the cos ϕ was required to obtain the relationship between the electrical power consumed and the mechanical power output. Discuss what this means by relating the power consumed to the value of this multiplied by cos ϕ.

True power, apparent power and reactive power

The discussion at the end of the previous activity led to the conclusion that resistive power can be used to perform useful work, such as turning a motor shaft.

Reactive power contributes to the power consumption of the motor, but does not contribute to the useable power output.

The reactive power in the case of a motor is not available for useful power as it is used to create and sustain the magnetic field produced by the inductive component. Similarly, for circuits that contain capacitive components, the reactive power is used to create and sustain the electric field in the capacitor.

This leads to three types of power being defined for an AC circuit, each of which has its units written in a specific way to differentiate between them.

|	
True power, P	This is the power that can be used to perform useful work. Only the resistive components of power contribute to the true power. The units of true power are W (watts).
Reactive Power, Q	This is the power associated with the reactive components. To show that the power is not available for useful power it is given units is VAR (volt-amps reactive).
Apparent power, S	This is the total power consumed by the device in operation. It is the vector sum of the resistive and reactive powers. It is given units of VA (volt-amps).

Table 2
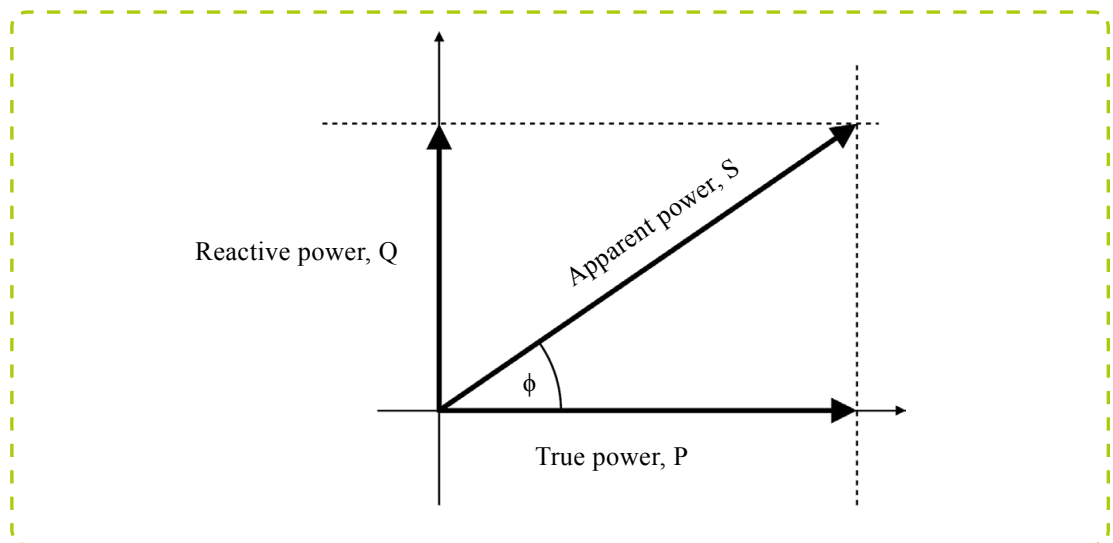
Figure 3
The relationship between true, reactive and apparent power.

Activity 5

1 Write an expression for S in terms of P and Q.

2 Write an expression for P in terms of S and the phase angle, ϕ.

3 Write an expression for Q in terms of S and the phase angle, ϕ.

4 Why is excessive reactive power undesirable?

Hint – Pythagoras and trigonometry

Power factor

In the last activity you found $P = S \cos\phi$, i.e. the true (useful) power is related to the apparent power (actually consumed by the device) through a multiplication by the power factor, $\cos\phi$, where ϕ is the phase angle between the voltage and the current.

Because of this relationship, it is easy to see that power factors near 1 (as the angle approaches zero) are desirable as more of the power consumed is available for useful power output.

Conversely, a low power factor means that a device consumes a relatively large amount of power during operation, but only a small fraction of this power is available for useful power output.

Activity 6

Knowing the power output from a motor, its efficiency and power factor is enough information to calculate the apparent power that a power source must be capable of delivering.

1 A different motor is rated at 375 W output, has an efficiency of η = 75% and a power factor of $\cos\phi$ = 0.7. Calculate the power supply required.
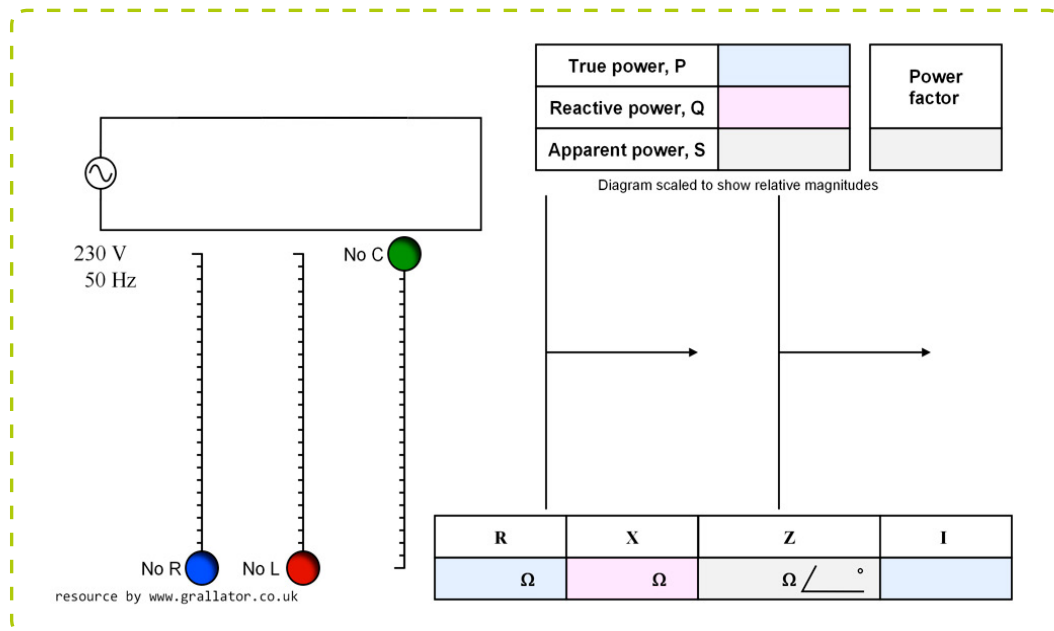
Figure 4
Screen shot of resource

The resource **lcr-series-2** shows an AC circuit that can have resistive, inductive and capacitive components.

2 NOTE, this is not the same motor as Activity 1! Use the resource to verify the impedance, current draw and true, reactive and apparent powers for the motor in this example (R = 100 Ω, L = 220 mH, no capacitance).

Stretch and challenge activity

Capacitive reactance contributes in the opposite sense to inductive reactance in a circuit. This means that the power factor of the motor can be improved by adding a capacitor to it. This is called adding a power factor correction.

1) Use the resource **lcr-series-2** to determine what standard size capacitor could be added in series to maximise the power factor.

2) Assuming the motor efficiency is unchanged, what is the output power produced by the motor now?

3) What is the current draw?

4) What might be the consequences of the addition of the capacitor?

5) What further change could you make to the circuit in order to yield an output power as close as possible to the original 250 W?

6) What effect does the change you make have on the originally rated current of 1.89 A? and what is the power saving?

Notes and solutions

Activity 1

Type	The manufacturer's model number.
Serial No.	A unique identifier for the unit.
Phase	The value 1~ means this is a single phase AC motor.
Volts	The supply voltage, in this case 230 V (rms).
Hz	The supply AC frequency, in this case 50 Hz.
Amps	The current drawn by the motor, in this case 1.89 A.
cos ϕ	This is called the power factor, it represents how much of the power supplied to the motor can be used to perform useful work. This is the item of interest in this activity. The power factor for this motor is 0.82.
Eff %	The efficiency with which the motor can turn useable input power to output power. This needs some explanation as the term 'useable' is defined by the power factor. The efficiency of this motor is 70%. Efficiency is usually denoted by the Greek symbol η.
kW	The useful power output of the motor, i.e. the power that can be transmitted to the motor output shaft. In this case, the motor can supply 0.25 kW of mechanical power.

The unit uses 230 V, 50 Hz single phase AC, which is the standard UK domestic supply.

Activity 2

1 The power consumed is given by

$$P = IV = 1.89 \times 230 = 434.7 \text{ W}$$

2 If this were converted to useful mechanical output power with an efficiency of η = 70%, the expected output power rating would be

$$P_{output} = P \times \frac{70}{100} = 434.7 \times 0.7 = 304.29 \text{ W}$$

This is larger than the stated output on the nameplate of 0.25 kW = 250 W. One possibility is that the manufacturer is being conservative with the design, i.e. it could produce about 300 W of mechanical power, but erring on the side of caution, the manufacturer stamps 250 W on the side to ensure the motor easily meets design requirements. In fact this is not the case. You will notice that if you multiply the result above by the power factor, denoted as cos ϕ on the motor, you arrive at a power that is about 250 W:

$$P_{output} = P \times \frac{70}{100} = 0.82 = 434.7 \times 0.7 \times 0.82 = 249.52 \text{ W} \quad (2 \text{ d.p.})$$

where $(P \times \dfrac{70}{100} = \text{Input} \times \eta)$ and $(0.82 = \cos \phi)$

The power factor (cos ϕ), then, is an extra correction on the conversion of power consumed by the motor to useful mechanical power at the output shaft.

Activity 3

1 $X_L = 2\pi fL = 2 \times \pi \times 50 \times 220 \times 10^{-3} = 69.115\ \Omega$ (3 d.p.).

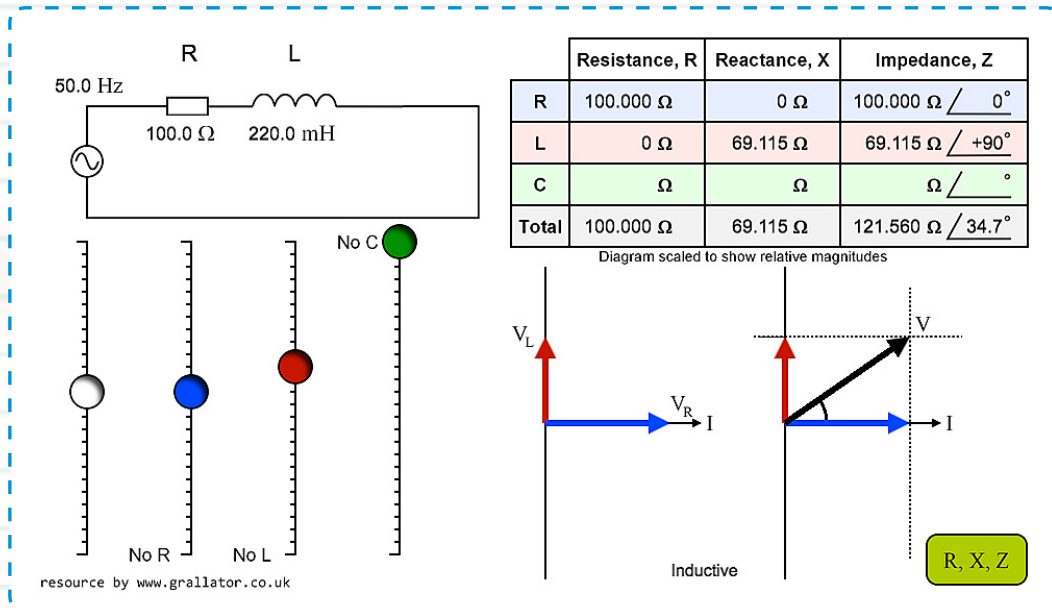**2** The full set of required values is shown below (using **lcr-series-1**).

	Resistance, R	Reactance, X	Impedance, Z
R	100.000 Ω	0 Ω	100.000 Ω ∠ 0°
L	0 Ω	69.115 Ω	69.115 Ω ∠ +90°
C	Ω	Ω	Ω ∠ °
Total	100.000 Ω	69.115 Ω	121.560 Ω ∠ 34.7°

Diagram scaled to show relative magnitudes

Figure 5
Screen shot of resource

3 Using $V = IZ$

$$230 = I \times 121.56$$

$$I = \frac{230}{121.56} = 1.892\ A \text{ (3 d.p.)}$$

The value on the motor has been rounded to 2 d.p.

Activity 4

1) The resistive power is given by $P_R = IV_R$. The reactive power is given by $P_X = IV_X$.

2) Using $V_R = IR$ and $V_X = IX$, using I rounded to 1.89 A and X rounded to 69.115 Ω gives

$$V_R = IR = 1.89 \times 100 = 189 \text{ V}$$

$$V_X = IX = 1.89 \times 69.115 = 130.627 \text{ V} \quad (3 \text{ d.p.})$$

So that

$$P_R = IV_R = 1.89 \times 189 = 357.21$$

and

$$P_X = IV_X = 1.89 \times 130.766 = 246.886 \quad (3 \text{ d.p.})$$

Note, that digits have been kept in the calculator during the above. Also note that I and X have been rounded (as has Z in the original calculation of I, see Activity 3) – keeping the full digits for these two values will result in $P_R = 357.991$ and $P_X = 247.426$. Note, units deliberately removed!
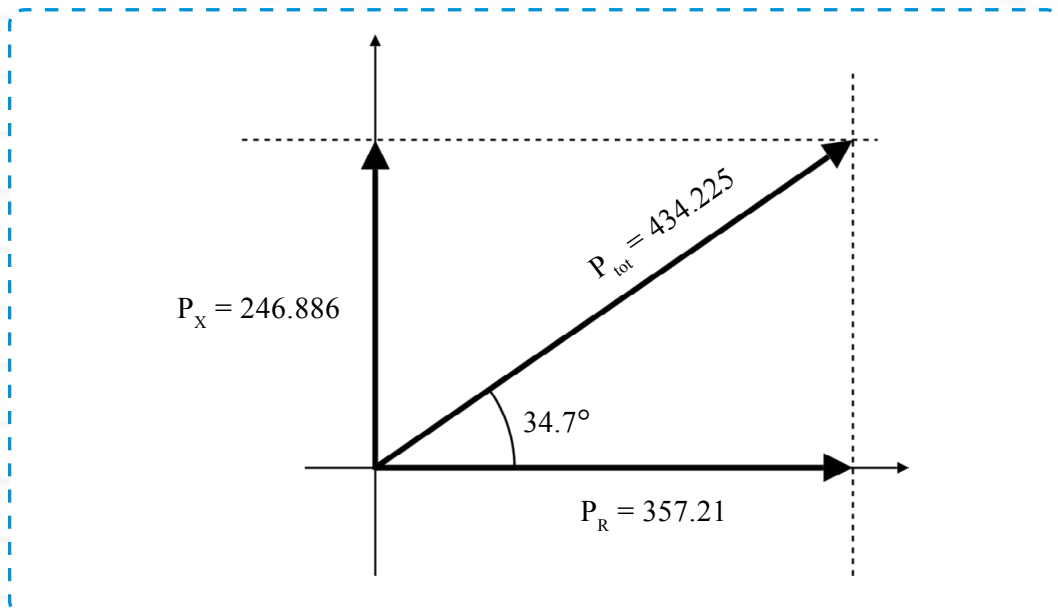
The power phasor is shown in **Figure 6**.

Figure 6
Power phasor diagram

The vector sum of the components is about 434.2 compared with the power calculation in activity 2 of 434.7 (units deliberately missing). These are very close and are, in fact, due to calculation rounding. A calculation keeping all digits from all derived values would give identical values for the two calculations. Units will be covered in Activity 5.

3) The phase angle is ϕ = 34.7°. Finding the cosine:

$$\cos \phi = P_R/P_{Ttot} = \cos 34.7° = 0.82 \quad (2 \text{ d.p.})$$

This is the same value stamped on the motor.

4 The mechanical power output is related to the electrical power consumed through

$$P_{out} = P_{consumed} \times efficiency \times \cos \phi$$

The efficiency tells you how effective the motor is at converting electrical power to mechanical power. Looking at the other terms, and relating them to the power phasor diagram you can see by trigonometry that $P_{consumed} \times \cos \phi = P_R$, i.e. only the resistive component of the power can be used to provide useful power out. The reactive component does not contribute to the power output.

Activity 5

1 Using Pythagoras $S = \sqrt{P^2 + Q^2}$.

2 Using trigonometry $P = S \cos \phi$, i.e. the true (useful) power is related to the apparent power (actually consumed by the device) through a multiplication by the power factor, $\cos \phi$.

3 Using trigonometry $Q = S \sin \phi$.

4 Reactive power is undesirable in a circuit as it increases the power that must be provided to produce a given useful true power output, making operation more expensive.

Activity 6

1 Slide R and L to the appropriate values, ensure C is set at 'No C'. The resource will show values that are slightly different from the calculations in Activity 4 as all digits are kept in all derived values.

2 The output is given by the product of the true power, P, and efficiency, $P_{out} = P \times eff$, so the true power is

$$P_{out} = P \times eff$$

$$P = \frac{P_{out}}{eff} = \frac{375}{0.75} = 500 \text{ W}$$

To calculate the apparent power, S, use $P = S \cos \phi$,

$$P = S \cos \phi$$

$$S = \frac{P}{\cos \phi} = \frac{500}{0.7} = 714.29 \text{ VA}$$

Note the units.

Stretch and Challenge

1 A series capacitor of 47 µF in series with the motor will give a power factor of 1.000 (3 d.p.) and the following values for P, Q and S

True power, P	528.898 W
Reactive Power, Q	7.349 VAr
Apparent power, S	528.949 VA

2 The power output of the motor with an efficiency of 70% is now

$$528.898 \times \frac{70}{100} = 370.2 \text{ W} \ (1 \text{ d.p.})$$

3 The current draw is 2.3 A. This is an increase of 0.41 A from the previous value of 1.89 A. This is to be expected as the voltage remains unchanged at 230 V, but the impedance has been reduced from 121.560 Ω to 100.010 Ω.

4 The power output has been increased by the addition of the capacitor, but so has the current drawn. The increased power output may put excessive load on the motors components leading to damage and increased unreliability. Further, the increased in the current drawn may be too high for the conductors and components used in the motor leading to damage. Also, the power supply may not be designed to deliver the new current.

5 The current drawn can reduced by increasing the resistance in the circuit. To obtain an output power of 250 W, with an efficiency of η = 70%, requires a true power of

$$P = \frac{250}{\frac{70}{100}} = \frac{250}{0.7} = 357.1 \text{ W}$$

Increasing the total resistance to 150 Ω (which can be achieved practically by adding a resistor in series in the circuit) gives a true power of 352.6 W, which is close to the desired value of 357.1. The power output for in this configuration will be $352.6 \times \frac{70}{100} = 246.8 \text{ W}$, which is a reduction of only about 1% in power.

6 The current drawn with the increased resistance is now 1.533 A. This is a current reduction of about 19%, which is, remember, achieved with only about a 1% reduction in output power. The apparent power has reduced from about 435 VA to about 353 VA, representing about a 19% saving in the power consumed.

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