AC phasors and fault detection

Topic areas

Electrical and electronic engineering:

- AC theory
- Resistance, reactance and impedance

Mathematics:

- Phasors
- Vector addition
- Pythagoras' theorem
- Trigonometry

Prerequisites

None

Problem statement

When AC power is distributed and used in an electrical power application there will usually be a monitoring system in place to detect any faults that will inevitably develop through prolonged, long-term use.

How can engineers use the characteristics of an AC circuit to detect when a fault condition has occurred?

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Activity 1 – Discussion

There are many kinds of faults that could develop in an AC system, for example, a short circuit.

Think about an electrical transmission line feeding a transformer that reduces the voltage for domestic use. What events could cause a fault to develop?

Background

AC circuits can contain elements that are resistive or reactive.

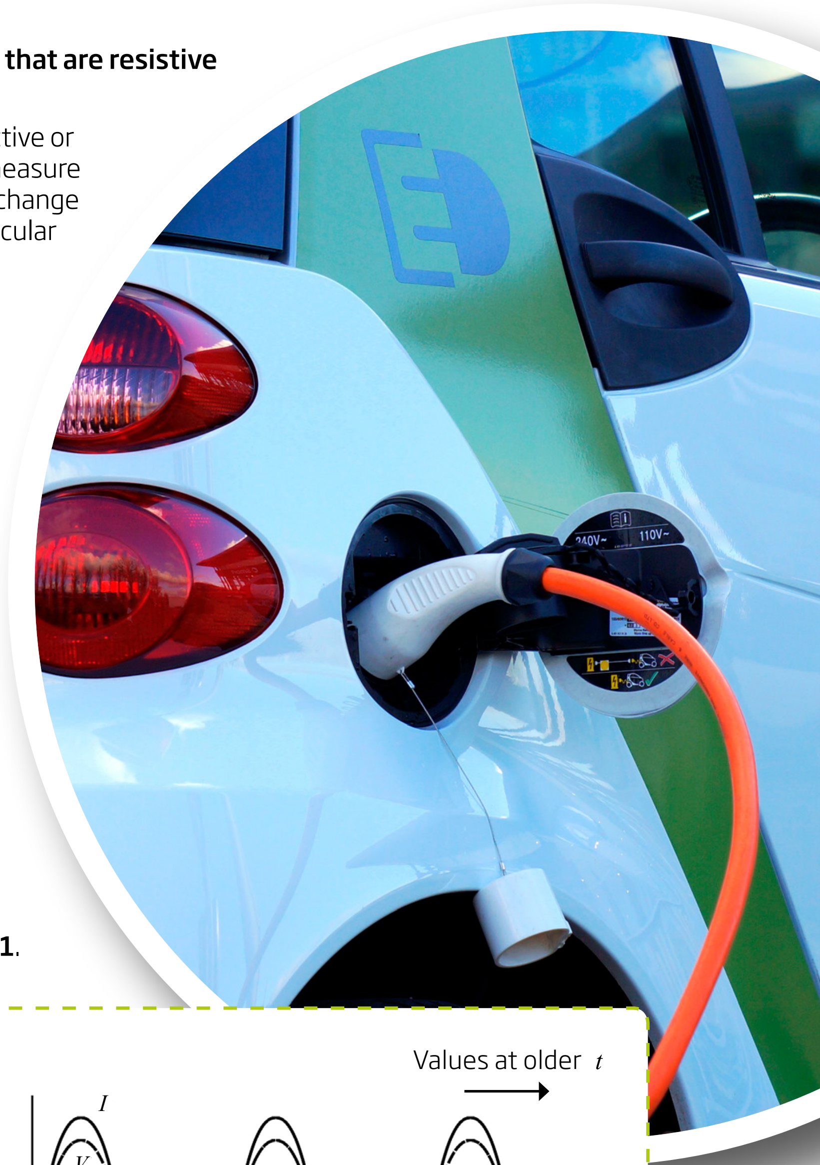
The reactive elements can be inductive or capacitive, and the *reactance* is a measure of the opposition in the circuit to a change in current or voltage. This is of particular relevance to AC, as the current and voltage are constantly changing.

In most practical systems there are elements that are inductive or capacitive. For example, motors use wire coils and so introduce inductance.

The same motor may also contain a start capacitor to improve start-up performance.

This will become apparent as you go through the resource. The combined resistance, R, and reactance, X, of an AC circuit is called the impedance, Z.

In a purely resistive circuit the voltage and the current are in phase, meaning that the peak voltage occurs when the peak current occurs, as shown in **Figure 1**.



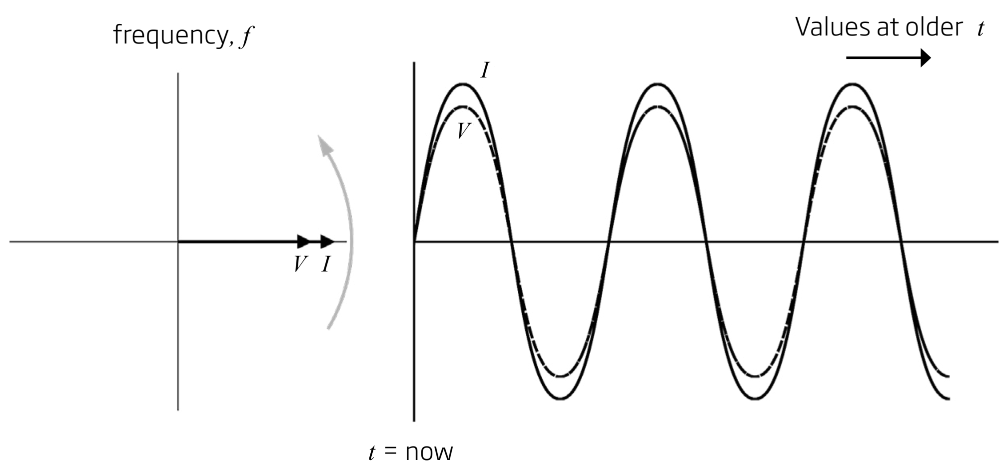


Figure 1
Phasor diagram

However, when the circuit also has reactance, the peak voltage does not necessarily occur at the same time as the peak current. This can be seen in **Figure 2**. In this example, the voltage leads the current due to reactance effects in the circuit, which will be discussed in this resource. The relationship between voltage and current is described by a *phasor*, which is a rotating depiction of the change in voltage and current. The angle between the voltage and current in the diagram is called the *phase angle*.

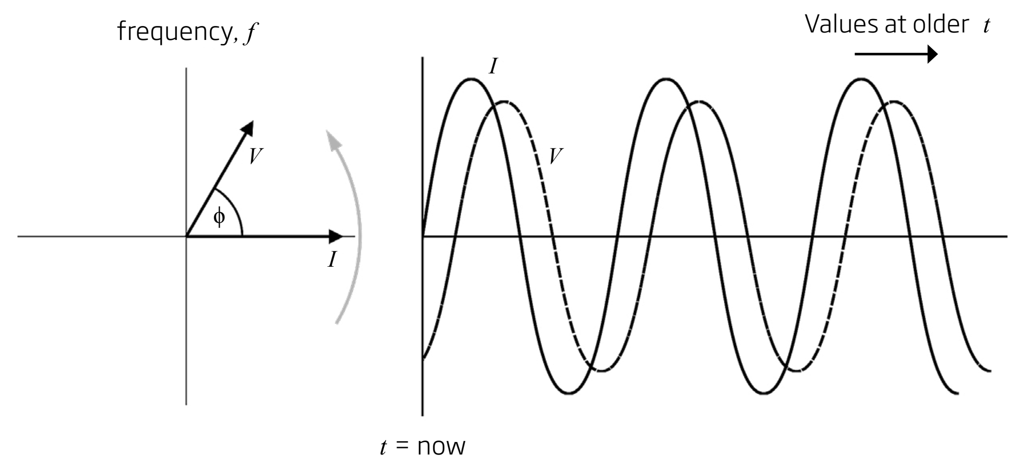


Figure 2
Phasor diagram

When working with AC circuits, it is convenient to use a phasor diagram, which is a snapshot of the rotating phasor, usually at a time when the current is pointing horizontally to the right. The left hand graph above is a phasor diagram.

Activity 2 – Interactive

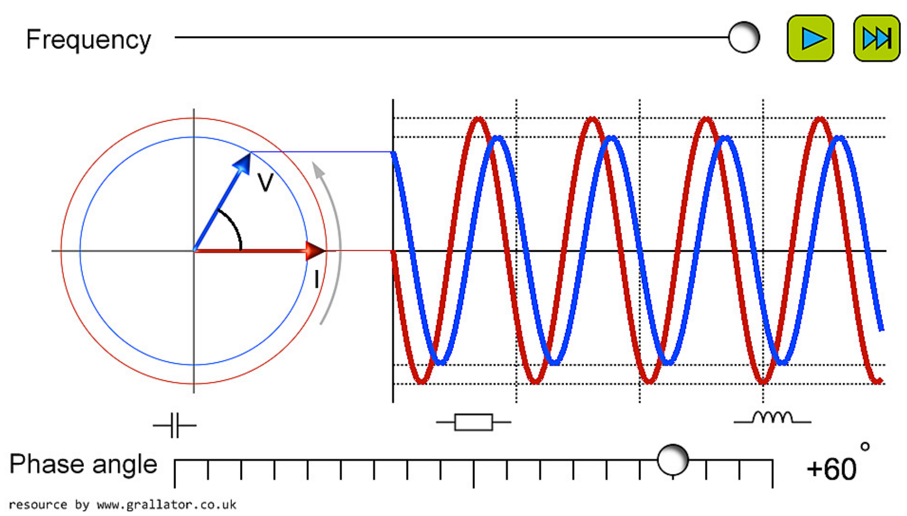


Figure 3
Screenshot of resource

The resource **phasor-1** shows an animated phasor diagram on the left, with the resulting voltage and current traces shown in the animation on the right, as seen in **Figure 3**. Experiment with the phase angle and frequency, and discuss what you notice for the following:

1. As the phase angle changes from representing purely inductive circuit (+90°), through a purely resistive circuit (0°), to a purely capacitive circuit (-90°).
2. As the frequency changes.

Relationship between resistance, reactance and impedance

Resistance and reactance can be plotted as vectors on a phasor diagram.

The resistance is plotted along the x-axis, parallel with the current to show that there is no phase difference between the current and voltage in a purely resistive circuit.

The inductive reactance is plotted vertically upwards, as a purely inductive phase angle is +90°, while the capacitive reactance is plotted vertically downwards, as a purely capacitive phase angle is -90°.

The total impedance of all the components combined, Z, is given by the vector sum of the components, as shown in **Figure 4**.

The impedance is stated as a value in Ohms, followed by a phase angle, for example, $60\Omega\angle30°$.

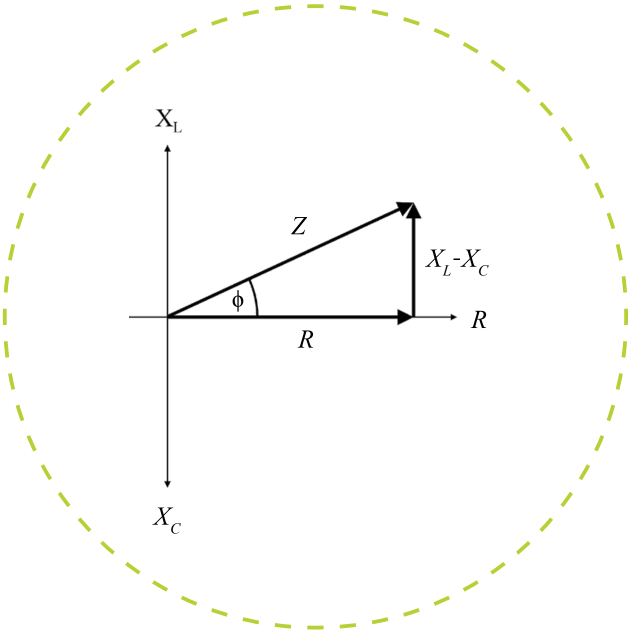


Figure 4
Relationship between resistance, reactance and impedance

Activity 3 - Interactive

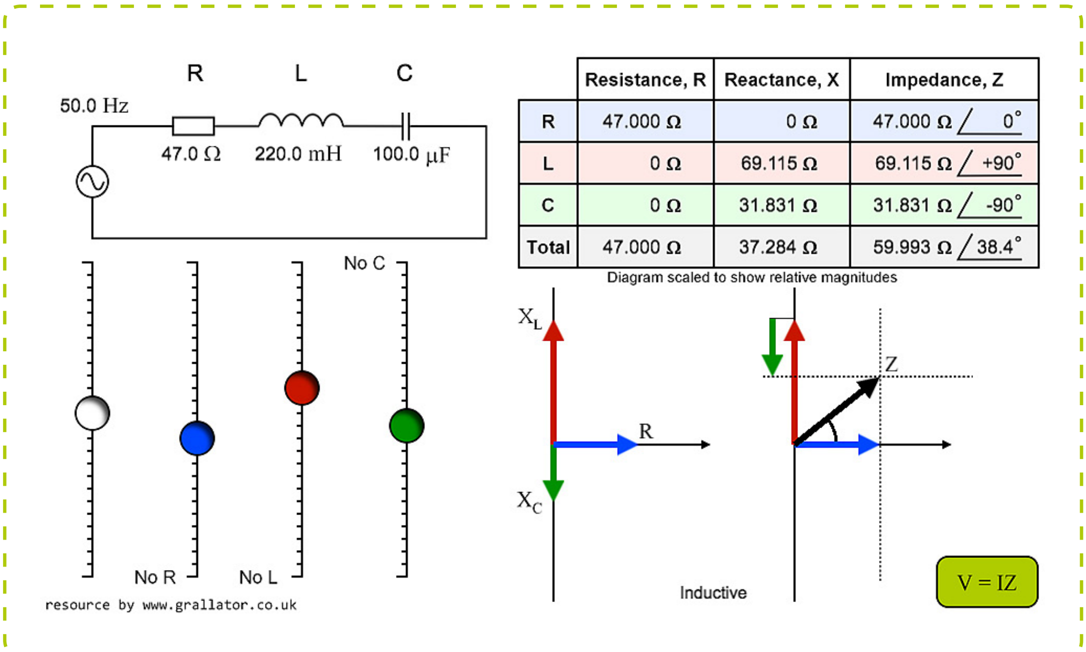


Figure 5
Screenshot of resource

The resource lcr-series-1 shows an AC circuit that can have resistive, inductive and capacitive components.

1) Experiment with the different values, and also change the AC frequency. What behaviour do you notice?

The resistance in an AC circuit is simply given as R. The reactance values due to an inductor of inductance, L, and capacitor of capacitance, C, are given in **Table 1**.

Inductor	Capacitor
$X_L = 2\pi f L$	$X_C = \dfrac{1}{2\pi f C}$

Table 1

Notice that the reactance values are dependent on the frequency of the AC, given by f.

2) Verify, by calculation, the reactance values in the resource screenshot above.

3) What do you notice about the nature and value of total reactance as X_L and X_C are changed?

4) How do the expressions in **Table 1** explain the behaviour you see when you experiment with the values in the resource, especially when you change the frequency?

5) Look at the two graphs shown in **Figure 6**. Which plot shows the behaviour of an inductor and which shows the behaviour of a capacitor?

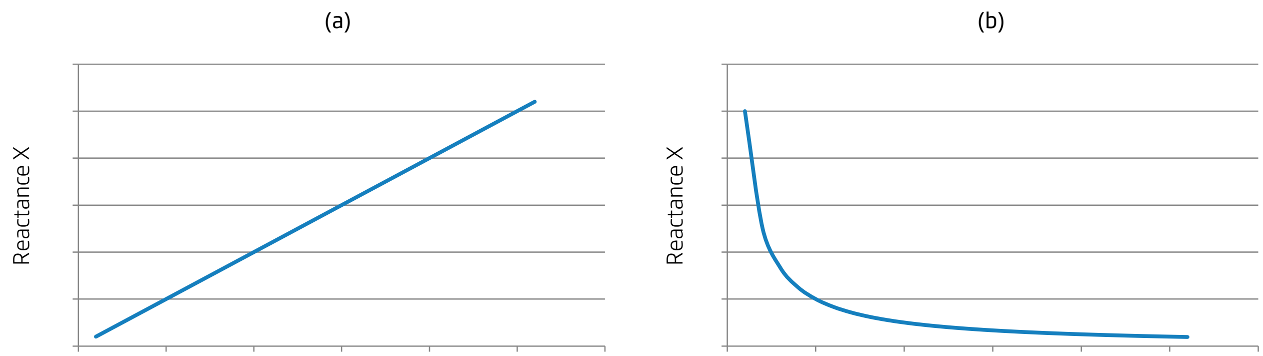


Figure 6

Ohm's law for an AC circuit uses impedance rather than resistance, so that $V = IZ$, rather than $V = IR$. As Z consists of a magnitude and an angle, $V = IZ$ not only gives you the relationship between the magnitude of V and I ($V/I = Z$), but also the phase angle between the two (it is the same as the angle given in the impedance). Click the $V = IZ$ button in the resource to show this.

Stretch and challenge activity

Look at the diagram shown in Figure 7 (taken from the resource lcr-series-1).

Using Pythagoras, write an expression for the magnitude of impedance Z, in terms of R, X_L and X_C. Write a separate expression for the phase angle.

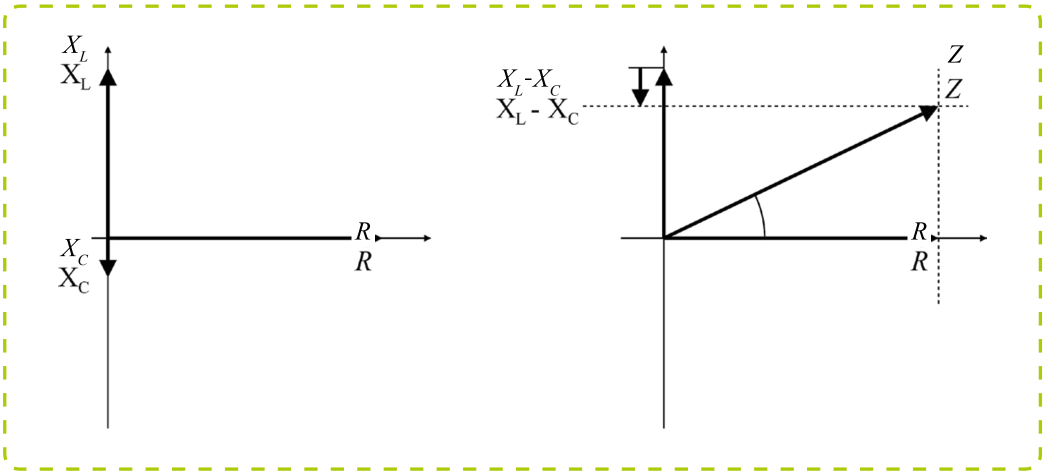


Figure 7
Relationship between resistance, reactance and impedance

Verify that when

R = 47 Ω

L = 220 mH

C = 100 μF

f = 50 Hz

The impedance is $59.993\Omega\angle 38.4°$

This can easily be used for voltages around an AC circuit. The circuit is inductive as X_L-X_C is positive.

Activity 4 – A circuit under normal conditions and under fault conditions

A circuit in normal operation can be depicted as a power transmission line connected to a load.

The transmission line and load will have characteristic resistance, inductance and capacitance values, as shown in **Figure 8**.

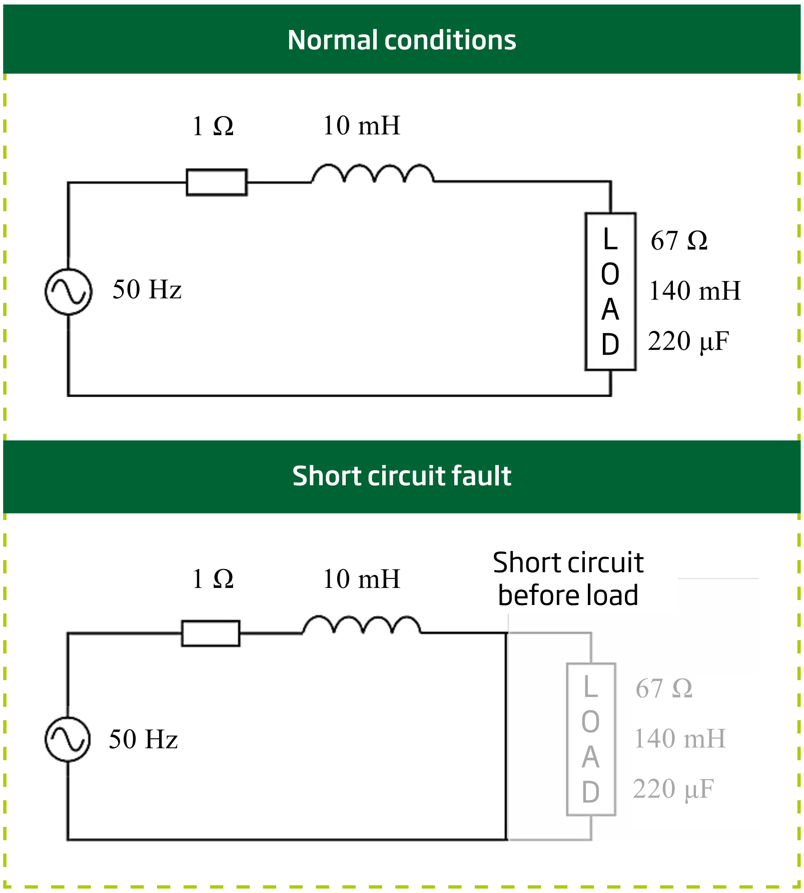


Figure 8 Characteristic values of a circuit in normal and fault conditions

Calculate the reactance values for the circuit under normal and fault conditions.

Use the results to draw a phasor diagram for each of the conditions and measure the phase angle with a protractor. Use the resource **lcr-series-1** to calculate the phase angle to verify your results.

1. How could this be used to remotely detect a fault?
2. Discuss where you would set a limit for fault detection.

Hint – this is a series circuit, so the total resistance is given by the sum of all the resistance values, and similarly, the total inductance is the sum of all the inductance values. There is only one capacitance value, which is only in the circuit during normal conditions.

Notes and solutions

Activity 1

There are many possible causes of faults, some of which include:

- Trees falling or growing and touching the wires.
- Lightning striking the equipment and causing damage.
- An accident, such as a fire, that affects the equipment or insulation.
- A general failure of the equipment due to use.
- Vandalism.

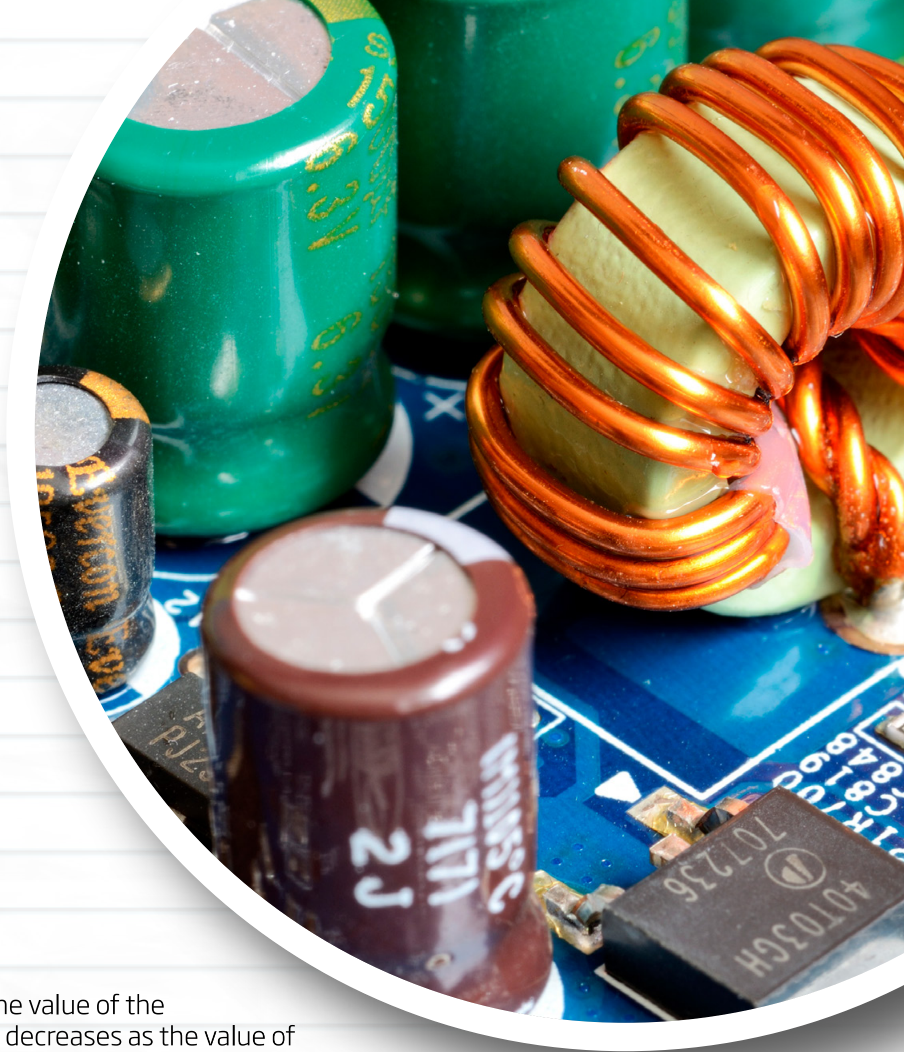
Activity 2

1. When the circuit is purely inductive, the phase angle is +90°. This means that the voltage leads the current by 90° – the blue voltage trace peaks before the red current trace.

   As the circuit becomes closer to a purely resistive one, for example as the phase angle approaches 0°, the voltage and current phases line up. As the circuit becomes more capacitive, the voltage starts to lag behind the current – the blue voltage trace peaks after the red current trace.

2. As the AC frequency increases, the phasor rotates more rapidly and the peaks in the traces move closer together.

   As the AC frequency decreases, the phasor rotates more slowly, and the peaks in the traces move further apart.



Activity 3

1. The following may be observed:
   - The total impedance increases as the value of the resistor and inductor increases, but decreases as the value of the capacitor increases.
   - The reactance of the inductor increases as AC frequency increases, but the reactance of the capacitor decreases.
   - The value of the resistance is independent of the frequency.
   - It is possible for the reactance of the inductor to be cancelled by an equal and opposite reactance from the capacitor, yielding a circuit with a phase angle close to zero.
   - When resistance, inductance and capacitance are all present in the circuit, then, for a given set of R, L and C values, the impedance will be high at high and low frequency values, and there will be a frequency between these at which the impedance will be minimal.

2. $X_L = 2\pi fL = 2 \times \pi \times 50 \times 220 \times 10^{-3} = 69.115\,\Omega$

   $$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times \pi \times 50 \times 100 \times 10^{-6}} = \frac{1}{3.142 \times 10^{-2}} = 31.831\,\Omega$$

   Don't forget to take account of the units-prefix of inductance (mH) and capacitance (μF).

3. The total reactance is the absolute (positive) difference between X_L and X_C.

4. Experimentation shows that the inductive reactance increases as inductance or frequency increases. This is expected as increased values of L or f increases $X_L = 2\pi fL$. Similarly, the capacitive reactance decreases as the capacitance or frequency increases. Again, this is expected as increased C or f increases the value of $2\pi fC$, so decreases the value of $X_C = \dfrac{1}{2\pi fC}$.

5. The graph (a) on the left is for an inductor. The graph (b) on the right is for a capacitor.

Stretch and challenge

The impedance is the length of the hypotenuse of a right-angled triangle of other side lengths R and $(X_L - X_C)$, as shown in **Figure 9**.

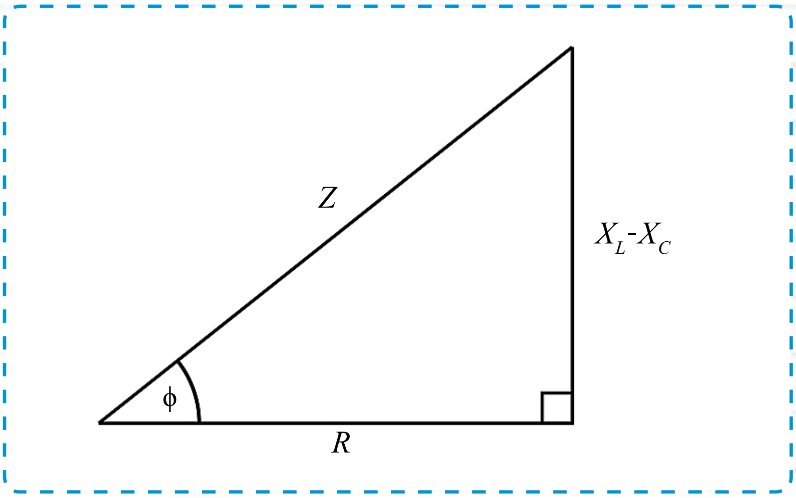


Figure 9
Relationship between resistance, reactance and impedance

By Pythagoras, the magnitude of Z is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$

Notice that because the difference in reactance terms is squared, it doesn't matter whether the magnitude of X_L is less that X_C in the above. The phase angle is given by $\tan\phi = \frac{X_L - X_C}{R}$

When X_L is smaller than X_C, the numerator is negative and the phase angle is negative, as expected.

Stretch and challenge - *continued*

When

$R = 47\ \Omega$
$L = 220$ mH
$C = 100\ \mu$F
$f = 50$ Hz

$$X_L = 2\pi fL = 2\times\pi\times 50\times 220\times 10^{-3} = 69.115\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\times\pi\times 50\times 100\times 10^{-6}} = \frac{1}{3.142\times 10^{-2}} = 31.831\ \Omega$$

the magnitude of the impedance is (keeping all digits in the calculator)

$$\begin{aligned} Z &= \sqrt{R^2 + \left(X_L - X_C\right)^2} \\ &= \sqrt{47^2 + \left(69.115 - 31.831\right)^2} \\ &= 59.993 \end{aligned}$$

the phase angle is

$$\tan\phi = \frac{69.115 - 31.831}{47} = \frac{37.284}{47} = 0.793$$

so that

$$\begin{aligned} \phi &= \tan^{-1} 0.793 \\ &= 38.4° \end{aligned}$$

the impedance is therefore $59.993\Omega\angle 38.4°$.

Activity 4

Normal conditions	Fault conditions
Total resistance = 1 + 67 = 68 Ω	Total resistance = 1 Ω
Total inductance = 10 + 140 = 150 mH $X_L = 2\pi fL = 2\times\pi\times 50\times 150\times 10^{-3} = 47.124\ \Omega$	Total inductance = 10 mH $X_L = 2\pi fL = 2\times\pi\times 50\times 10\times 10^{-3} = 3.142\ \Omega$
Capacitance = 220 μF $X_C = \frac{1}{2\pi fC} = \frac{1}{2\times\pi\times 50\times 220\times 10^{-6}} = 14.469\ \Omega$	No capacitance $X_C = 0$
Total reactance = 32.655 Ω	Total reactance = 3.142 Ω
The phase angle for this circuit is 25.7°	The phase angle for this circuit is 72.3°

The phasor diagrams are shown in **Figure 10**.

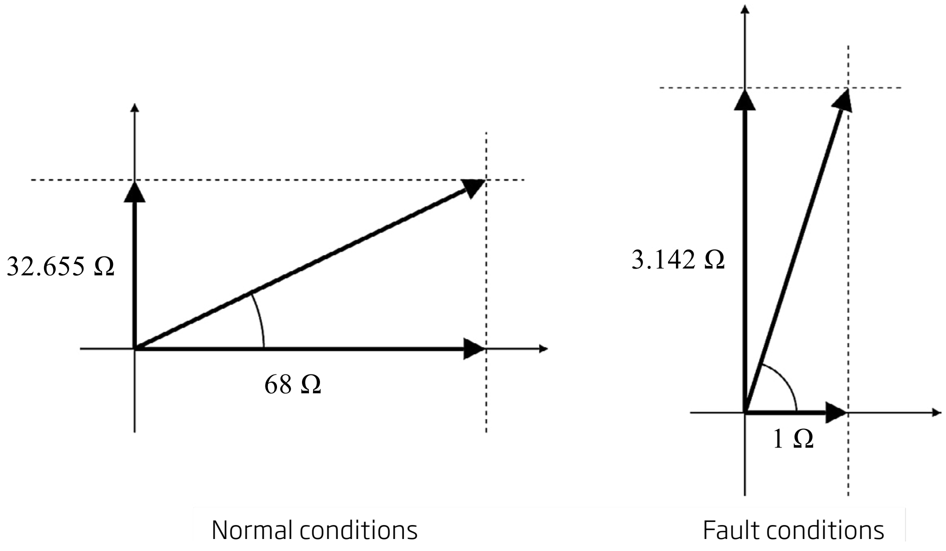


Figure 10
Phase angles under normal and fault conditions

The phase angle has increased from 25.7° under normal conditions, to 72.3° under fault conditions.

1. It is possible to measure the phase angle between the voltage and the current using a phase meter. The expected phase angle for the circuit under normal conditions is 25.7°. If the measured value changes to 72.3°, it is likely that a short circuit fault has occurred.

2. It might not be wise to set the fault detection at 72.3° as the meter might have measurement uncertainties associated with it.

   It might also be the case that the short circuit has some resistive and reactive characteristics instead of having none, as assumed here. In this case, the phase angle may not be as high as 72.3°.

   A practical fault phase angle should therefore be set somewhere between 25.7° and 72.3°.

   It should not be too low, otherwise it may falsely report a fault, when in reality there is none, and the change in phase angle may be due to measurement uncertainty, or the load has changed, which changes the phase angle by an allowable amount.

   Neither should the fault angle be too large as the nature of the fault might result in the predicted ideal amount of change in the phase, as calculated here. In this case, a fault will not be flagged when in fact there is one.

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