Co-planar forces on a crane

Mechanical engineering

- Direct forces
- Concurrent co-planar forces
- Force diagrams
- Resultant of a set of co-planar forces
- Conditions for equilibrium and motion
- Newton's second Law, $F = ma$

Mathematics

- Vector addition
- Resolving a vector into two perpendicular components
- Trigonometric functions for sine and cosine
- Trigonometric identities
- Simultaneous equations

Prerequisites

None

Problem statement

Mechanical design engineering is the basis of every engineered structure, whether static (non-moving) or dynamic (moving). One very important aspect is the calculation of the forces acting within and upon a structure and using these to ensure any design will be able to withstand the stresses and strains expected of it without failure.

For example, in the crane image, the cables must be strong enough to lift and move the bucket, while the rigid mechanical frame must be able to support the load without buckling. Additionally, the whole crane must be stable so that it does not tip over when manoeuvring a full bucket. How can the forces acting on a structure be determined and how do they change when an object that is initially stationary starts to move?

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Activity 1 – Discussion

The image given in the problem statement shows a drag line excavator. The bucket is supported and moved by two cables and can hold typically 100 to 200 tonnes of material. **Figure 1** shows a simplified view of a drag line bucket from the image on the front page.

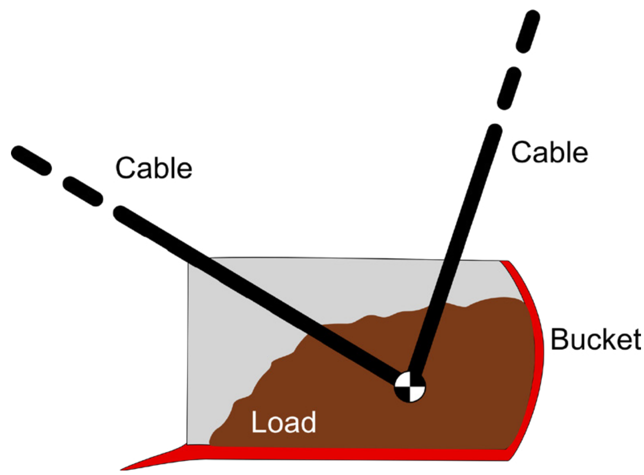
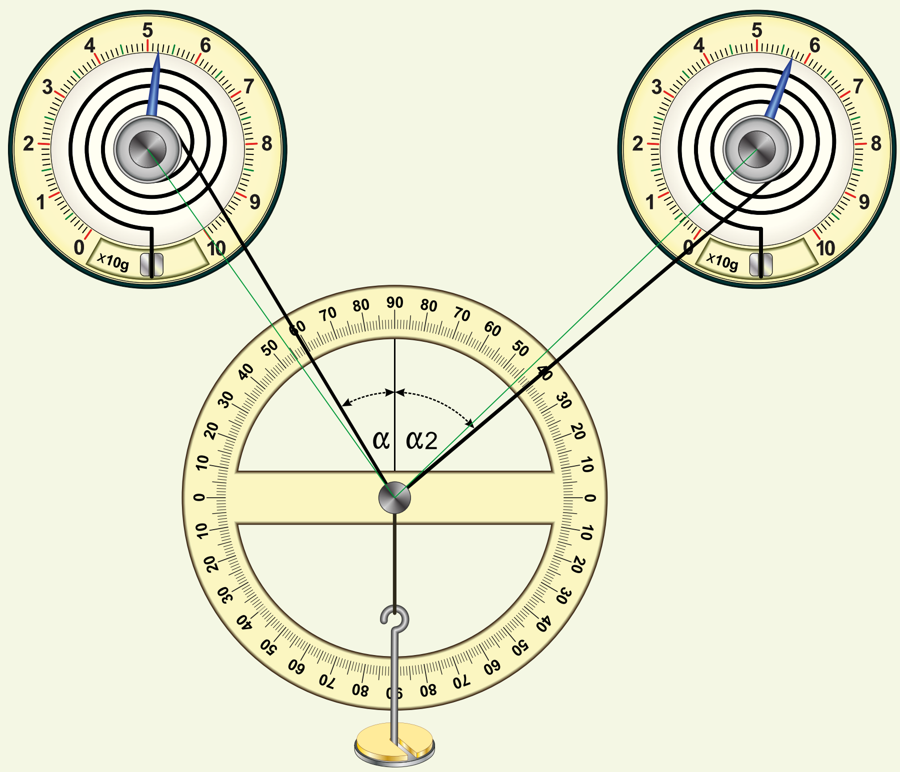


Figure 1
Simplified diagram of a drag line bucket

1 What forces are acting in the diagram and where?

Hint: Think, for example, about tension and weight.



Background – conditions for equilibrium and non-equilibrium

In **Figure 1** it is assumed that three forces are acting in the same plane (they are *co-planar forces*) and the geometry is such that they are acting through a single point (they are *concurrent forces*). When the bucket in **Figure 1** is stationary there are no net forces acting, for example all the forces balance. If there were a net force acting then, through Newton's second law, $F = ma$, the bucket would accelerate in the direction of the force. The net force can be found by joining the force vectors 'tail to head' as shown in **Figure 2**. There is no net force if this vector sum forms a closed path.

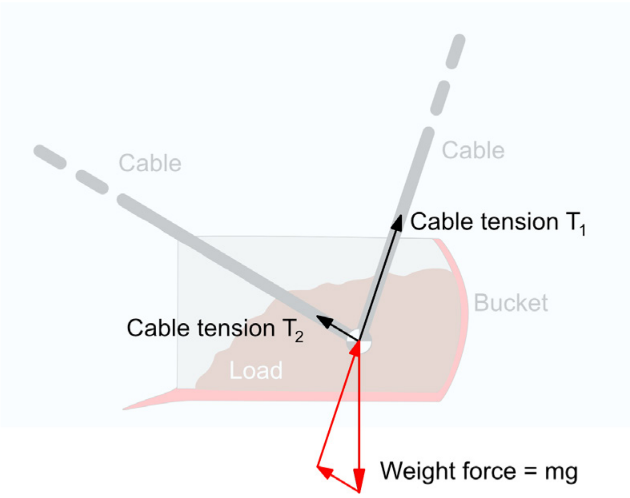


Figure 2
Vector sum of forces

When the excavator starts to move the bucket, there is an initial acceleration in the direction of motion and so there must therefore be a net force acting in the direction of motion. If the bucket accelerates to a maximum allowable speed, the acceleration falls to zero and there is no net force. However, the bucket still moves at a constant speed; zero force means no acceleration, not necessarily no motion.

The resource **vectorforceaddition.html** can be used to demonstrate and test knowledge of the vector addition of forces that are represented as a column vector and determine whether they are in equilibrium or not. The number of forces can be changed between 3 and 5. The 'Show resultant' button demonstrates 'tail-to-head' addition to generate a resultant. There is a 50% chance that any given problem will produce a zero-force resultant.

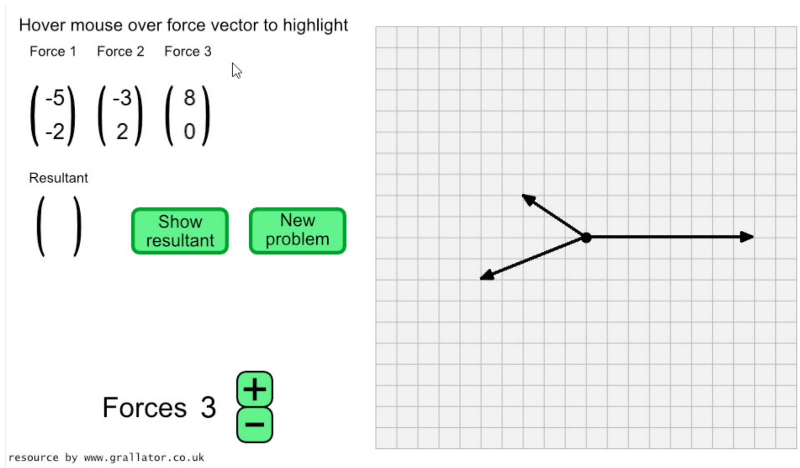


Figure 3
Screen shot of interactive resource

An additional activity using the resource could be to determine the magnitude of the force from its components.

Background - resolving the forces

The last activity considered force vectors that were given as horizontal and vertical components. This makes it easy to add up the components to produce a resultant, and the magnitude of the force can be found by using Pythagoras' theorem. However, there are problems where you are working with force magnitudes and angles and you need to construct the components. Doing this is called resolving the forces, and the trigonometric functions sine and cosine are used, as shown in **Figure 4**.

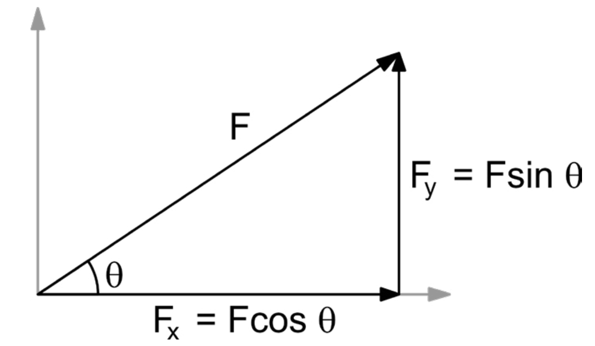


Figure 4
Resolving forces

Aside: To correctly form vector components you need to assign the correct sign to the values:

- Forces to the right are positive, forces to the left are negative.
- Forces up are positive, forces down are negative.

However, when a good diagram is drawn it is often more intuitive to work with positive values and put all the forces acting to the right on one side of an '=' sign and all the forces acting left on the other. Similarly, put all the forces acting up on one side of an '=' sign and all the forces acting down on the other. If, when solving a problem, you end up with a negative value, it means you have the direction of the force on the wrong side of the equation, for example you've assumed a force acts to the left, when in fact it acts to the right.

The resource **resloveforces.html** can be used to demonstrate and practise the resolving of forces into horizontal and vertical components. *Note, this resource assumes positive values of the components and the engineer is taking account of their direction on a diagram.*

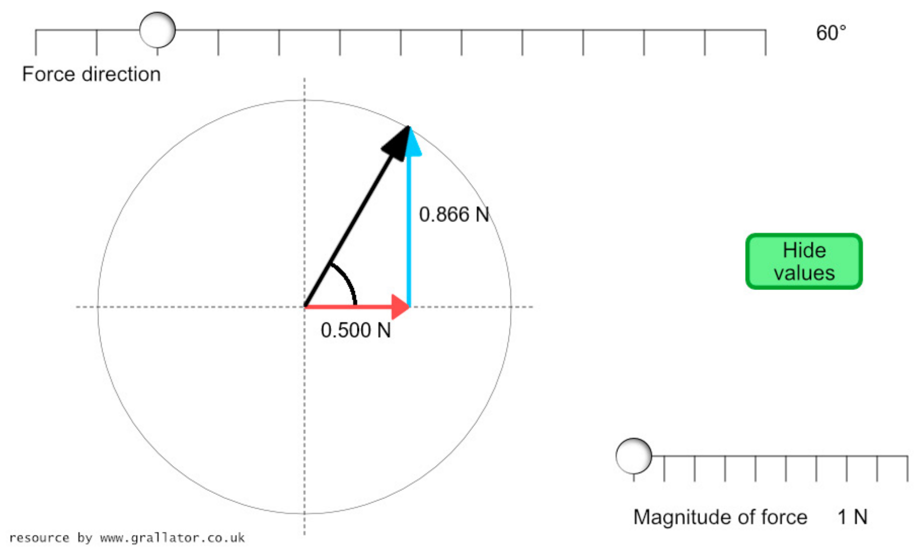
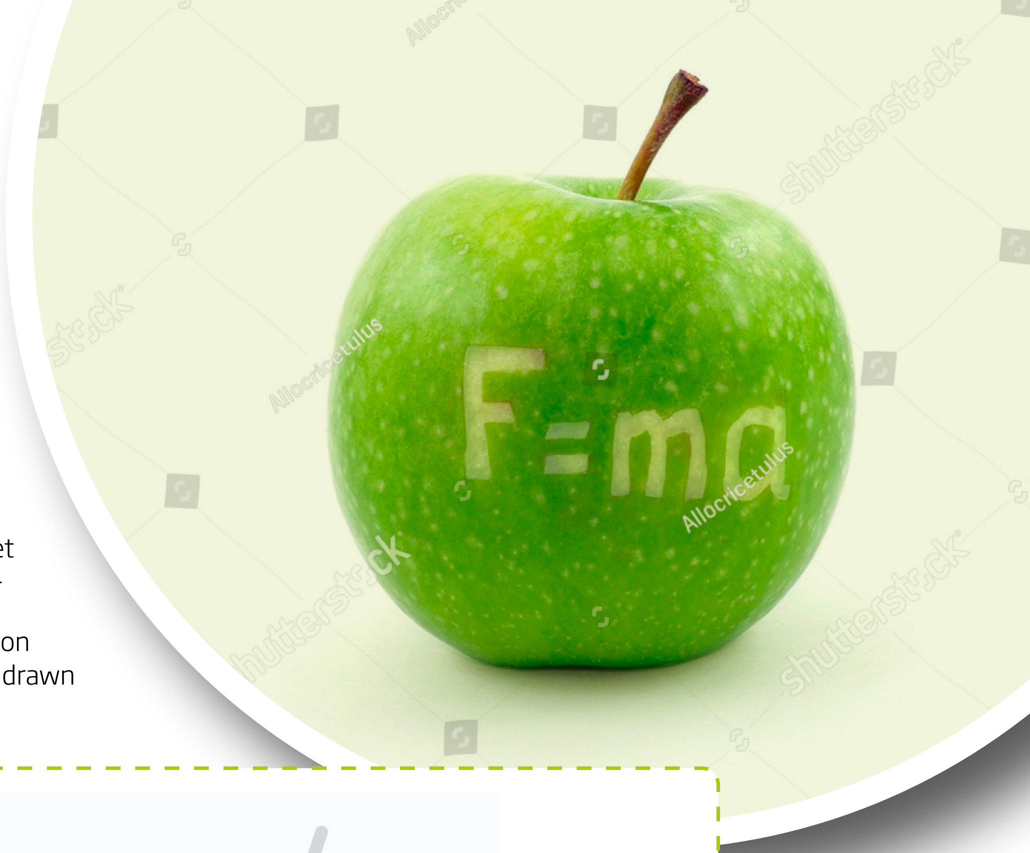


Figure 5
Screen shot of interactive resource



Activity 2 - Find the forces

Figure 6 shows a drag line bucket with a total weight of 1.4 MN (1.4 x 10^6 N), which is held in a static position by two cables with tension forces T_1 and T_2 (force diagram is drawn to scale).

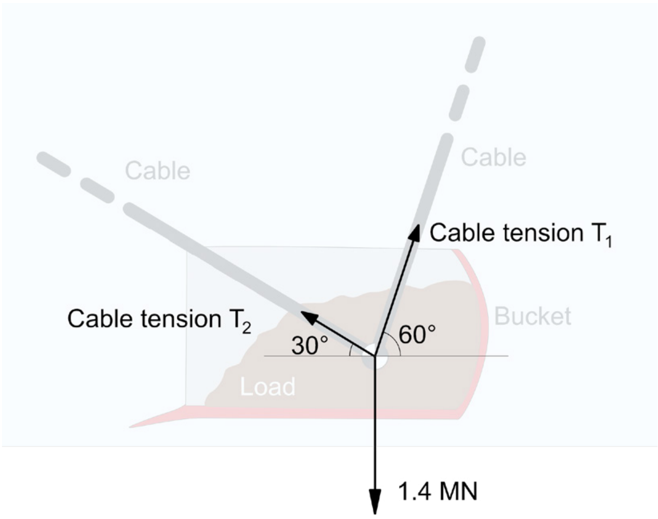


Figure 6 Forces acting on a drag line bucket

1. Which tension force do you predict will be greater T_1 or T_2, or do you think they will be the same? Give a reason for your prediction.
2. Construct an equation for the horizontal forces acting in the diagram.
3. Construct an equation for the vertical forces acting in the diagram.
4. Solve the two equations to calculate the force tensions T_1 and T_2. Compare the results with your prediction.

In your calculations use the following:

$\sin 30° = \frac{1}{2}$	$\sin 60° = \frac{\sqrt{3}}{2}$
$\cos 30° = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{1}{2}$

Activity 3 – Experiment and interactive resource

Fasten a relatively heavy weight to the middle of a piece of strong string. Hold each end of the string in each hand, starting with your hands close together so that the weight is directly below both hands. Pull your hands apart to change the angle the strings make from near vertical to a more horizontal position.

1. What do you notice about the force you have to apply to the ends of the string as you move your hands apart and make a shallower angle with the horizontal? Give a reason for your observation.

2. Can the string ever be horizontal?

The resource **ResolveForcesBucket.html** can be used to demonstrate and practise solving the tension problem for user-defined angles, and to give a theoretical version of the experiment described above. When the 'Scale vectors' button is green the vectors' forces are scaled relative to their magnitude and you can demonstrate equilibrium is achieved by clicking the 'Show vector sum' button. The 'Hide values' button allows the diagram to be shown without results for practice calculation purposes.

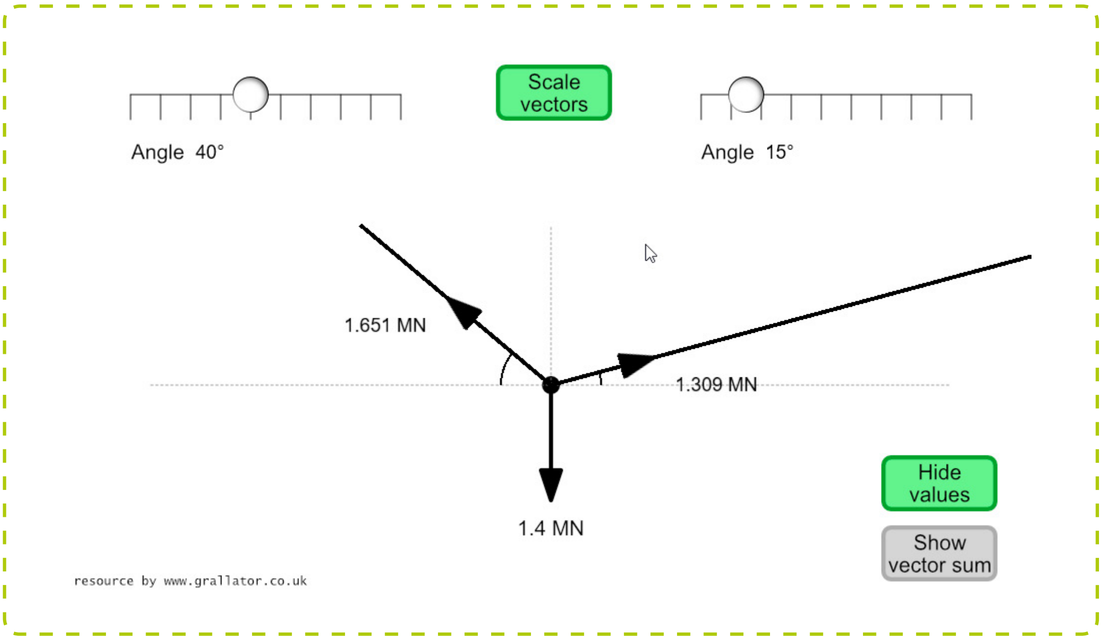


Figure 7 Screen shot of interactive resource

3. When holding the bucket above the ground the minimum angle the right-side cable is allowed to make with the horizontal is 15°, while the left-side cable is allowed to be horizontal. Use the interactive resource to calculate the tensions in the cables.

4. The cables are made of strands of 35 millimetre wire rope with a maximum safe load each of 138 kN. Calculate the minimum number of strands required for each of the cables for the stationary bucket case. Will this number of cables be sufficient during operation?

Stretch and challenge activity

The bucket shown in Activity 2, **Figure 6** is to be raised vertically upwards. Starting from rest it accelerates at 1 ms^{-2} until it reaches its safe raising speed of 0.2 ms^{-1}. Take g = 10 ms^{-2}:

1. Find the net force downwards on the bucket that produces an acceleration of 1 ms^{-2}.
2. Find an expression for the horizontal components of tension in the cables.
3. Find an expression for the vertical components of tension in the cables.
4. Solve the two equations to calculate the force tensions T_1 and T_2 at the initial instant of raising. Compare the results with your prediction.
5. Will the tensions remain the same as the bucket is raised?

Notes and solutions

Activity 1

1 The forces acting are shown in **Figure 9**.

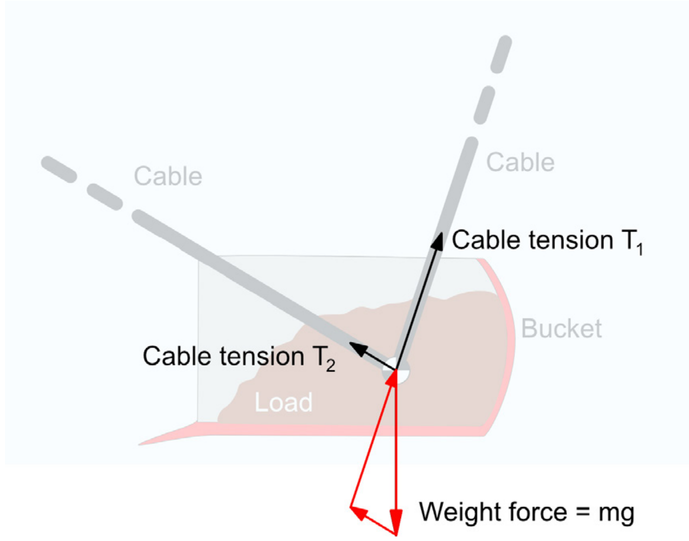


Figure 8
Forces acting on the bucket

Activity 2

1 Based on the force diagram (which is drawn to scale), the tension T_1 should be larger than the tension T_2.

2 Horizontally in static equilibrium, the forces acting to the right must be balanced by the forces acting to the left. This gives:

$$T_1 \cos 60 = T_2 \cos 30$$

$$\frac{T_1}{2} = \frac{T_2\sqrt{3}}{2}$$

$$T_1 = T_2\sqrt{3} \qquad (1)$$

3 Vertically in static equilibrium, the forces acting upwards must be balanced by the forces acting downwards. This gives:

$$T_1 \sin 60 + T_2 \sin 30 = 1.4$$

$$\frac{T_1\sqrt{3}}{2} + \frac{T_2}{2} = 1.4$$

$$T_1\sqrt{3} + T_2 = 2.8 \qquad (2)$$

4 Substitute T_1 from expression (1) into the expression (2) derived above to give:

$$T_1\sqrt{3} + T_2 = 2.8$$
$$T_2\sqrt{3} \cdot \sqrt{3} + T_2 = 2.8$$
$$3T_2 + T_2 = 2.8$$
$$4T_2 = 2.8$$
$$T_2 = 0.7 \text{ MN}$$

From which:

$$T_1 = T_2\sqrt{3} = 0.7 \times \sqrt{3} = 1.212 \text{ MN (3 d.p.)}$$

Activity 3

1 As you pull your hands apart to try to make the string horizontal the force required increases. To explain why, consider the vertical component of force as shown in **Figure 9**. As the string nears the horizontal, the angle θ becomes small on each side of the weight. This means that a smaller fraction of the total tension force applied is available to support the vertical weight force acting downwards. As the weight is constant, this means that the total tension force (F) you apply must increase to compensate.

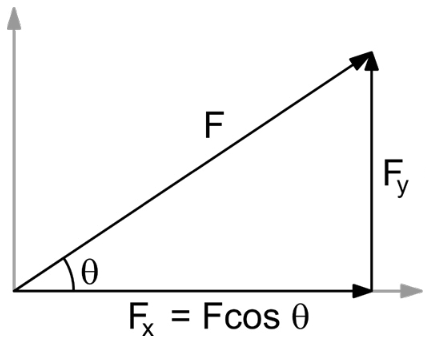


Figure 9 Components of a force

2 As the angle of the string approaches zero, the force required to support the weight becomes infinite. If the string were ever to be truly horizontal then there would be no vertical component acting upwards to counter the weight downwards, and the weight should not be in static equilibrium.

Note, if the string can stretch a little and the weight attached is small, you may be able to pull the string horizontal. However, if you look at the region where the weight is attached you will observe a slight downward bend in the string.

3 The interactive resource predicts T_1 (right cable) = 5.409 MN and T_2 (left cable) = 5.225 MN.

4 Each strand has a maximum safe load each of 138 kN = 0.138 MN. The number of strands required for each cable is, and remembering you should round up to the nearest whole number:

$$N_{right} = \frac{5.409}{0.138} = 40$$

$$N_{left} = \frac{5.225}{0.138} = 38$$

These numbers are valid only for the static case. When the bucket is lifted upwards there will be a net acceleration upwards that will increase the tension in the cables. The number of strands calculated above therefore is not sufficient for operation.

Stretch and challenge activity

1. There is a net acceleration upwards of 1 ms^{-2}, so using Newton's second law, $F = ma$, there must be a net force, F, upwards.

   The weight force of the bucket and load is 1.4 MN, which is given by $F = mg$, so its mass in kg (with g = 10 ms^{-2}) is $m = \dfrac{1.4 \times 10^6}{10} = 1.4 \times 10^5$ kg.

   Substituting for mass and acceleration gives:

   $$F = ma$$

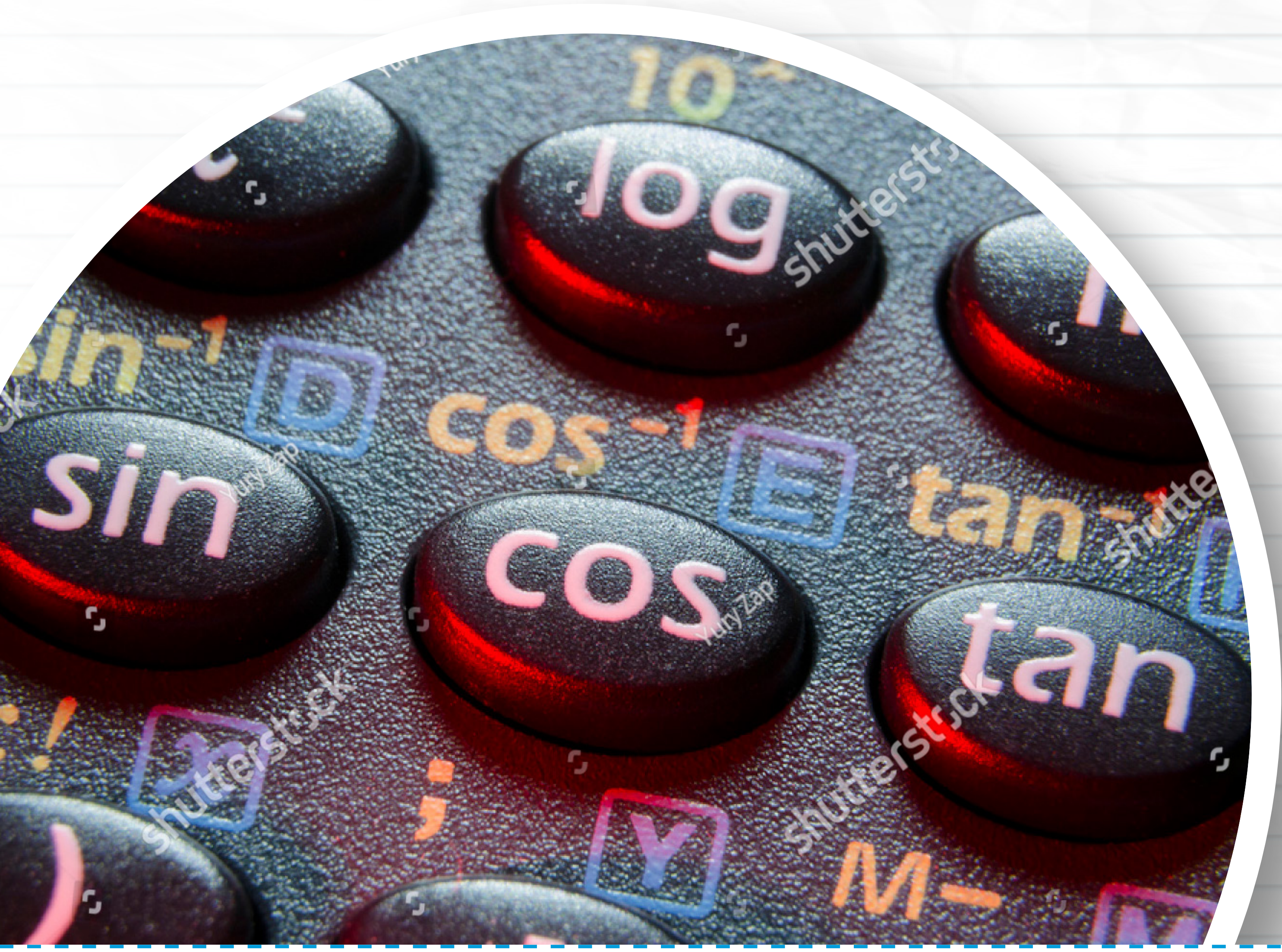
   $$F = 1.4 \times 10^5 \times 1 = 1.4 \times 10^5 \text{ N} = 0.14 \text{ MN}$$

2. As previously given, horizontally in static equilibrium the forces acting to the right must be balanced by the forces acting to the left. This gives:

   $$T_1 \cos 60 = T_2 \cos 30$$

   $$\frac{T_1}{2} = \frac{T_2\sqrt{3}}{2}$$

   $$T_1 = T_2\sqrt{3} \qquad (3)$$



3 Vertically, the system is not in static equilibrium. As there is a net acceleration upwards, the vertical component of tension acting upwards must be greater than the weight acting downwards and, as the mass and acceleration are given, the difference between the upwards and downwards forces is given by the value of F calculated above, so that:

$$T_1 \sin 60 + T_2 \sin 30 - 1.4 = F$$

$$T_1 \sin 60 + T_2 \sin 30 - 1.4 = 0.14$$

$$T_1 \sin 60 + T_2 \sin 30 = 1.4 + 0.14$$

$$T_1 \sin 60 + T_2 \sin 30 = 1.54$$

$$\frac{T_1\sqrt{3}}{2} + \frac{T_2}{2} = 1.54$$

$$T_1\sqrt{3} + T_2 = 3.08 \qquad (4)$$

4 Substitute T_1 from expression (3) into the expression (4) derived above to give:

$$T_1\sqrt{3} + T_2 = 3.08$$

$$T_2\sqrt{3} \cdot \sqrt{3} + T_2 = 3.08$$

$$3T_2 + T_2 = 3.08$$

$$4T_2 = 3.08$$

$$T_2 = 0.77 \text{ MN}$$

From which:

$$T_1 = T_2\sqrt{3} = 0.77 \times \sqrt{3} = 1.334 \text{ MN (3 d.p.)}$$

These values may be compared with $T_2 = 1.212$ MN and $T_2 = 0.7$ MN that were calculated for the stationary case in Activity 2.

5 The tensions will not stay the same as the bucket is raised for two reasons:

a. The cables will change angle to be less vertical as the bucket raises, which increases the tension.

b. The bucket only accelerates until the safe raising speed is reached, at which point it moves at a constant speed. As there is no further acceleration from this point there is again no net force upwards, which decreases the tension.

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