

# Filtering AC signals

## Topic areas

### Electrical and electronic engineering:

- ✓ AC Theory.
- ✓ Resistance, reactance and impedance.
- ✓ Potential divider in an AC circuit.
- ✓ Low pass and high pass filters.

### Mathematics:

- ✓ Vector addition.
- ✓ Pythagoras' theorem.
- ✓ Trigonometry.
- ✓ Phasor multiplication and division (stretch).

## Prerequisites

It may be useful to look at the resource '**AC phasors and fault detection**' to introduce phasor diagrams and the relationship between resistance, reactance and impedance.

## Problem statement

An alternating current is not necessarily associated solely with a power supply. Analogue signals from sensors and instruments can also be thought of as being AC signals, although, in contrast to a power supply, the frequency usually changes with time. An example is the recording and playing back of sounds using a microphone and speaker.

However, sometimes such signals are 'contaminated' with unwanted components, such as high-frequency microphone 'hiss', or low frequency mains 'hum', which you want to filter out to improve the sound quality. Another example could be that when playing back sounds, you want to select and direct low frequency sounds to a large bass speaker and higher frequency sounds to smaller 'tweeter' speakers to take advantage of the performance characteristics of each.

Yet another example is in the detection and decoding of radio communication signals, where the receiver selects a relatively narrow range of radio frequencies, then, in general, filters out the radio carrier signal to recover the original sound or data signal.

How can you select signals based on their frequency?

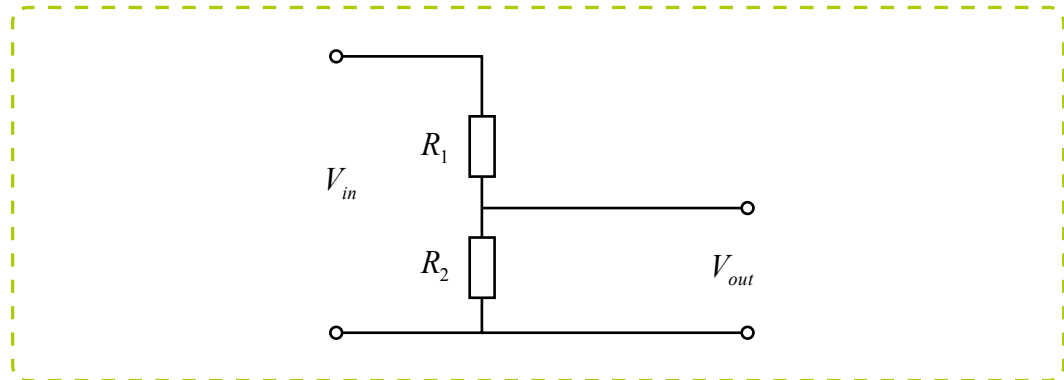


## Activity 1 - Discussion

Discuss some examples of where you might want to filter out high or low frequencies

## Background

The circuit below shows a potential divider circuit using two resistors



**Figure 1**  
A potential divider

The total resistance in the circuit on the left is given by  $R_{tot} = R_1 + R_2$ . Using Ohm's law, the current flowing through the resistors is related to the voltage across them,  $V_{in}$ , as

$$V = IR$$

$$V_{in} = IR_{tot}$$

$$I = \frac{V_{in}}{R_{tot}} = \frac{V_{in}}{R_1 + R_2}$$

As the resistors are in series, the same current must flow through each. Ohm's law can be used on each resistor to calculate the voltage drop across it (the sum of the voltage drops across each resistor must equal the total voltage drop,  $V_{in}$ ). The diagram shows that the voltage drop across  $R_2$  is denoted as  $V_{out}$ , so that

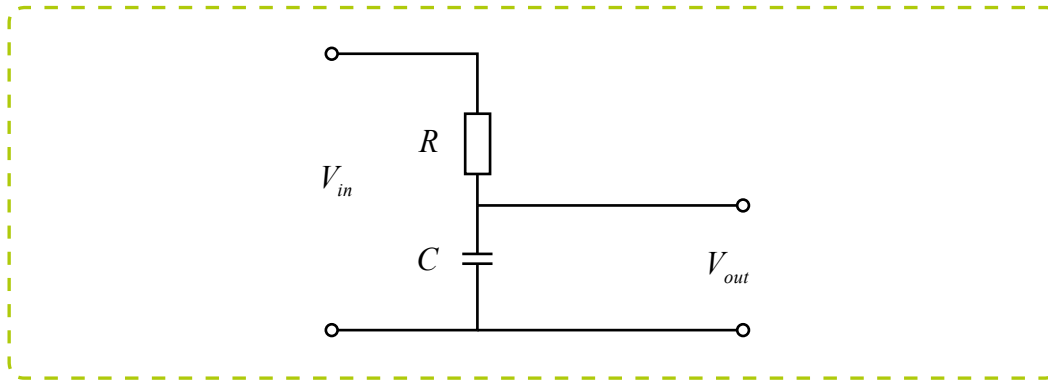
$$\begin{aligned} V_{out} &= IR_2 \\ &= \frac{V_{in}}{R_1 + R_2} \times R_2 \end{aligned}$$

Rearranging to give

$$\frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2}.$$

This gives the ratio of the magnitude of the output to the input. A small value means that the output voltage is small compared with the input voltage. A large value means that the output voltage is comparable with the input voltage.

## Activity 2 - Replacing a resistor with a capacitor



**Figure 2**  
A potential divider for AC signals using a resistor and a capacitor

**Figure 2** shows a potential divider for an AC circuit where  $R_2$  in **Figure 1** has been replaced by a capacitor. The reactance of a capacitor of capacitance  $C$  for an AC signal of frequency  $f$ ,  $X_c$  is given by

$$X_c = \frac{1}{2\pi fC}$$

- 1 Find the total impedance of the circuit in **Figure 2**,  $Z$ , when  $R = 1 \text{ k}\Omega$ ,  $C = 1 \text{ }\mu\text{F}$  and  $f = 100 \text{ Hz}$ .

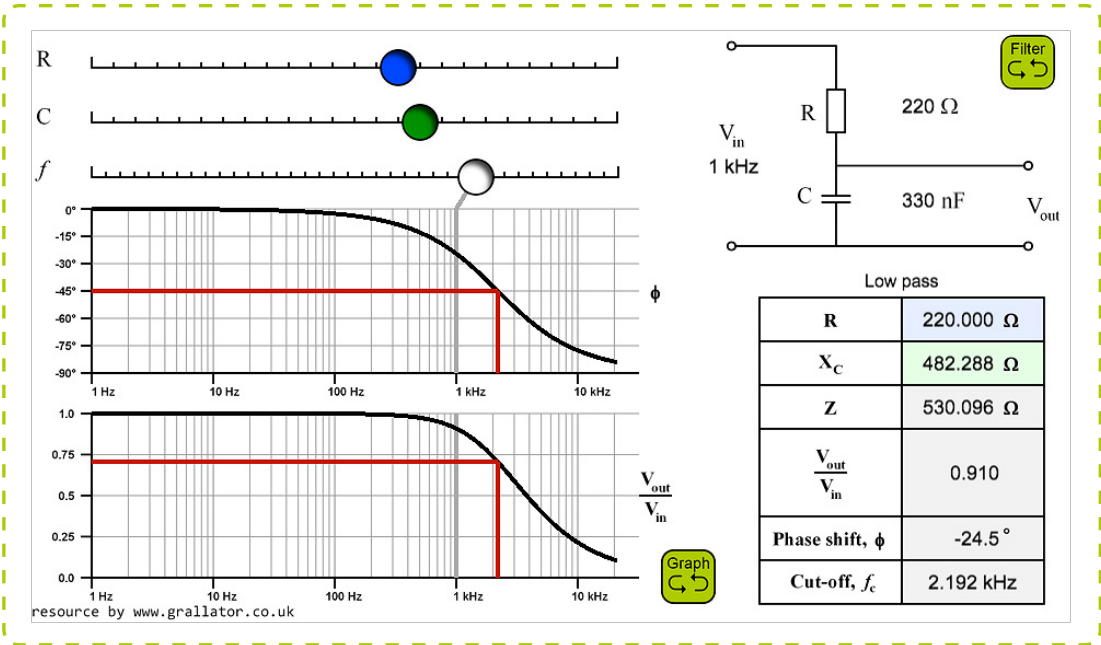
*Hint - Remember the impedance is calculated by finding the vector sum of the resistance and reactance, **See Resource AC Power**.*

- 2 Look at the expression for  $\frac{V_{out}}{V_{in}}$  in the background discussion above. Use this, and the expression you used to calculate the total impedance in question 1 above to write an expression for the ratio  $\frac{V_{out}}{V_{in}}$  in terms of  $R$  and  $X_c$ .



# Activity 3 - Interactive

The resource [filter-1](#) shows the circuit shown in **Figure 2** and plots its characteristics for a full range of signal frequencies from 1 Hz to 20 kHz on a logarithmic frequency scale. Additionally, you can slide the frequency slider to obtain a set of values of  $X_c$ ,  $Z$  and  $\frac{V_{out}}{V_{in}}$  for a particular frequency.



**Figure 3**  
Screen shot of  
resource

The lower plot shows  $\frac{V_{out}}{V_{in}}$  for a range of frequencies; a value of one means the frequency is passed, while a lower value shows increasing filtering. The table shows 'spot' values depending on the frequency marker value.

- 1** Use the resource [filter-1](#) to fill in **Table 1**. Discuss how changing the value of  $C$  affects the results.

|                          | Case a                                                                            | Case b                                                                          | Case c                                                                           |
|--------------------------|-----------------------------------------------------------------------------------|---------------------------------------------------------------------------------|----------------------------------------------------------------------------------|
|                          | $R = 1\text{ k}\Omega$ ,<br>$C = 0.1\text{ }\mu\text{F}$ ,<br>$f = 100\text{ Hz}$ | $R = 1\text{ k}\Omega$ ,<br>$C = 1\text{ }\mu\text{F}$ ,<br>$f = 100\text{ Hz}$ | $R = 1\text{ k}\Omega$ ,<br>$C = 10\text{ }\mu\text{F}$ ,<br>$f = 100\text{ Hz}$ |
| $X_c$                    |                                                                                   |                                                                                 |                                                                                  |
| $Z$                      |                                                                                   |                                                                                 |                                                                                  |
| $\frac{V_{out}}{V_{in}}$ |                                                                                   |                                                                                 |                                                                                  |

**Table 1**

- 2 Use the resource [filter-1](#) to fill in **Table 2**. Discuss how changing the value of  $f$  affects the results.

|                          | Case a                                                                            | Case b                                                                             | Case c                                                                            |
|--------------------------|-----------------------------------------------------------------------------------|------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|                          | $R = 1 \text{ k}\Omega$ ,<br>$C = 1 \text{ }\mu\text{F}$ ,<br>$f = 10 \text{ Hz}$ | $R = 1 \text{ k}\Omega$ ,<br>$C = 1 \text{ }\mu\text{F}$ ,<br>$f = 100 \text{ Hz}$ | $R = 1 \text{ k}\Omega$ ,<br>$C = 1 \text{ }\mu\text{F}$ ,<br>$f = 1 \text{ kHz}$ |
| $X_c$                    |                                                                                   |                                                                                    |                                                                                   |
| $Z$                      |                                                                                   |                                                                                    |                                                                                   |
| $\frac{V_{out}}{V_{in}}$ |                                                                                   |                                                                                    |                                                                                   |

**Table 2**

- 3 Look at the results in **Table 2**. Why could the circuit in **Figure 2** be described as a low pass filter?

The red line on the resource [filter-1](#) shows the frequency at which the magnitude of the output is about 0.707 times the magnitude of the input. In fact

it shows when the magnitude of the output is 0.707 times the magnitude of the input. Technically, this is the frequency at which the power in the output signal is half the power of the input signal, where the power in an AC signal is related to its root mean square (RMS) value (not covered in this resource).

- 4 A signal of frequency greater than 2.3 kHz is mixed with the desired signal. Find a pair of values for  $R$  and  $C$  which give a cut off frequency near this value. Is the pair of values unique?
- 5 Discuss whether the shape of the filter behaviour plotted by the resource (lower-left plot) is ideal?

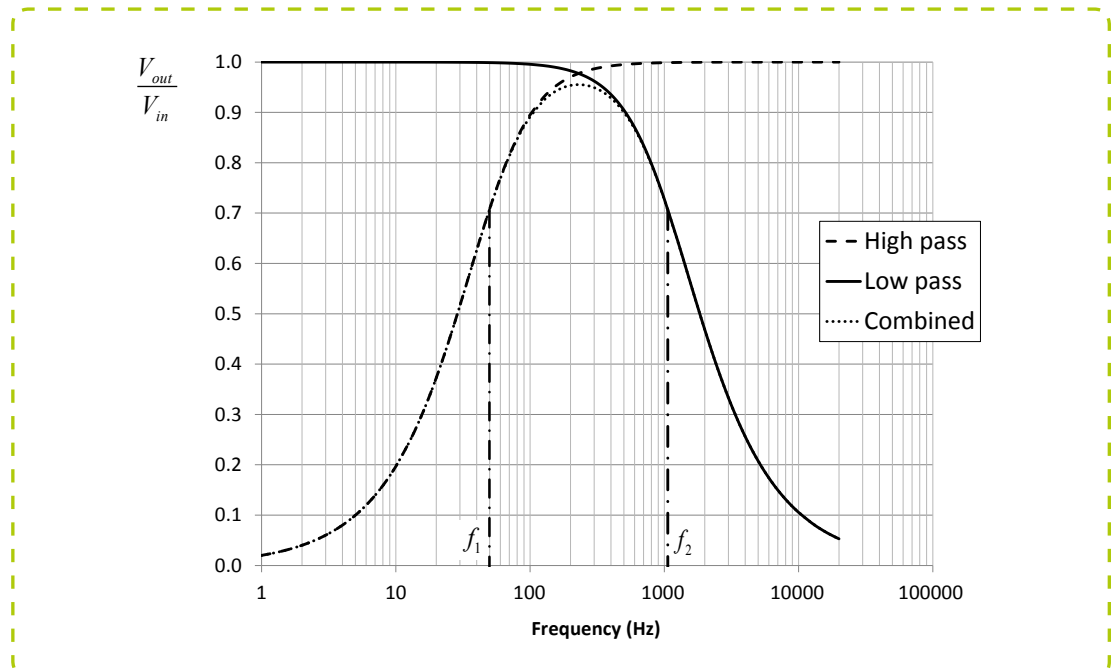
## Activity 4 - A high pass filter

Click the button marked  on the resource [filter-1](#). This swaps the position of the resistor and the capacitor. The filter now passes high frequency signals and filters low frequency ones, as shown by the lower plot.

- 1 What do you notice about the cut-off frequency of a high-pass filter and a low-pass filter when using the same  $R$  and  $C$  values?
- 2 Experiment with the values of  $R$  and  $C$  and observe how the cut off frequency is affected.
- 3 A mains hum of 50 Hz is being mixed with the signal. Find a pair of values for  $R$  and  $C$  which give a cut off frequency near this value. Is the pair of values unique?
- 4 Write an expression for the ratio  $\frac{V_{out}}{V_{in}}$  in terms of  $R$  and  $X_c$  for the circuit in the new configuration and explain why the circuit now filters out low frequencies.

## Background - Band pass filter

A signal that is first passed through a high pass filter with a cut off frequency  $f_1$  will cut off all frequencies less than  $f_1$ . Passing this filtered signal through a low pass filter with a cut off frequency of  $f_2$ , will cut off all the remaining frequencies greater than  $f_2$ . The combined effect is to pass a frequency band between  $f_1$  and  $f_2$ . Note, the graph below is a sketch only. A full analysis of circuit that combines a high pass filter with a low pass filter would need to consider how all the components interacted in a single circuit.



**Figure 4**  
A band pass  
filter

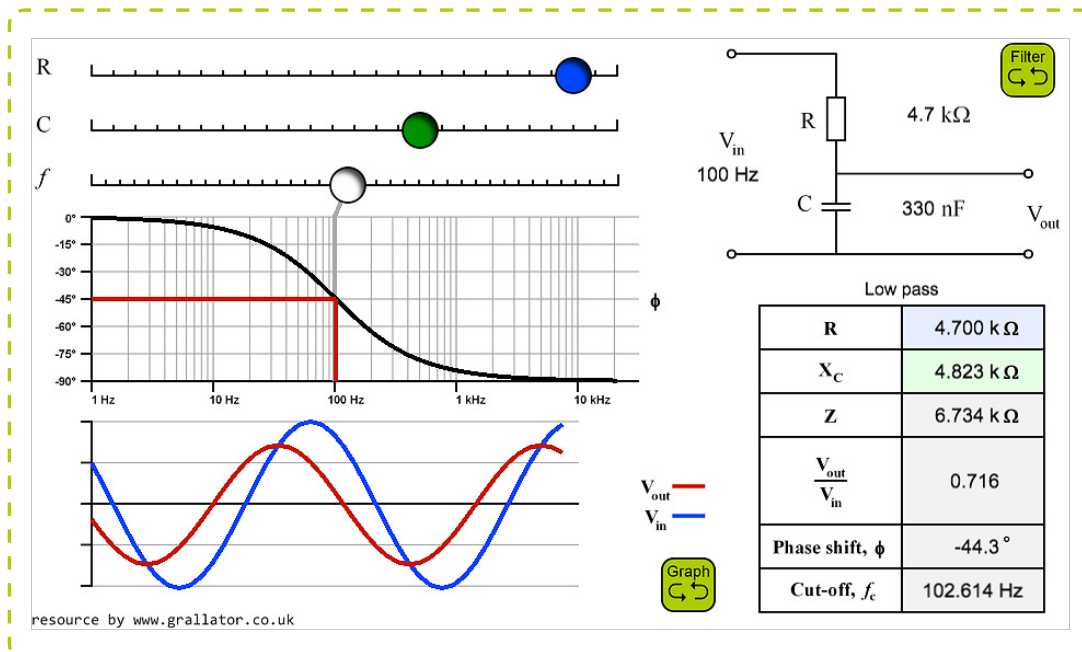
Such a filter is used in the detection and decoding of radio communication signals, where the receiver selects a relatively narrow range of radio frequencies to process to recover the original sound or data signal.

## Stretch and challenge activity

The use of reactive elements (capacitors and inductors) in a circuit changes the phase of the voltage with respect to the current, which passive components (resistors) do not. A change of phase is also seen in filter circuits, where the output signal is out of phase with the input signal. The phase angle is shown in the upper plot of the resource [filter-1](#). The amount of shift depends on the frequency of the signal. An animated view of the effect of the

phase shift can be seen by clicking the  button.





**Figure 5**  
Screen shot of  
resource

- 1 What do you notice about the phase shift for a low pass and a high pass filter as the frequency changes? What is the phase shift at the cut-off frequency?
- 2 The phase shift for a low pass filter is given by  $\phi = -\tan^{-1}(2\pi fCR)$ . Verify that the above configuration of  $R = 4.7 \text{ k}\Omega$ ,  $C = 330 \text{ nF}$  and  $f = 100 \text{ Hz}$  gives a phase shift of  $-44.3^\circ$ .
- 3 The phase shift for a high pass filter is given by  $\phi = \tan^{-1}\left(\frac{1}{2\pi fCR}\right)$ . Verify that using the above components in a high pass filter and signal of  $100 \text{ Hz}$  gives a phase shift of  $45.7^\circ$ .
- 4 Using phasor values for the resistance, reactance and impedance, and the expressions for  $\frac{V_{out}}{V_{in}}$  you found earlier verify that the angles in 2 and 3 above are correct by performing a phasor division.

*Hint - To multiply the phasor  $a\angle b^\circ$  by the phasor  $c\angle d^\circ$  you multiply the magnitudes and add the angles,*

$$a\angle b^\circ \times c\angle d^\circ = ac\angle(b+d)^\circ.$$

*To divide the phasor  $a\angle b^\circ$  by the phasor  $c\angle d^\circ$  you divide the magnitudes and subtract the angles,*

$$\frac{a\angle b^\circ}{c\angle d^\circ} = \frac{a}{c}\angle(b-d)^\circ.$$



# Notes and solutions

## Activity 1

A number of examples have been given in the introduction such as noise removal from a microphone, sending high or low frequencies to appropriate speakers for optimal sound output and frequency selection for radio.

**Other examples include**

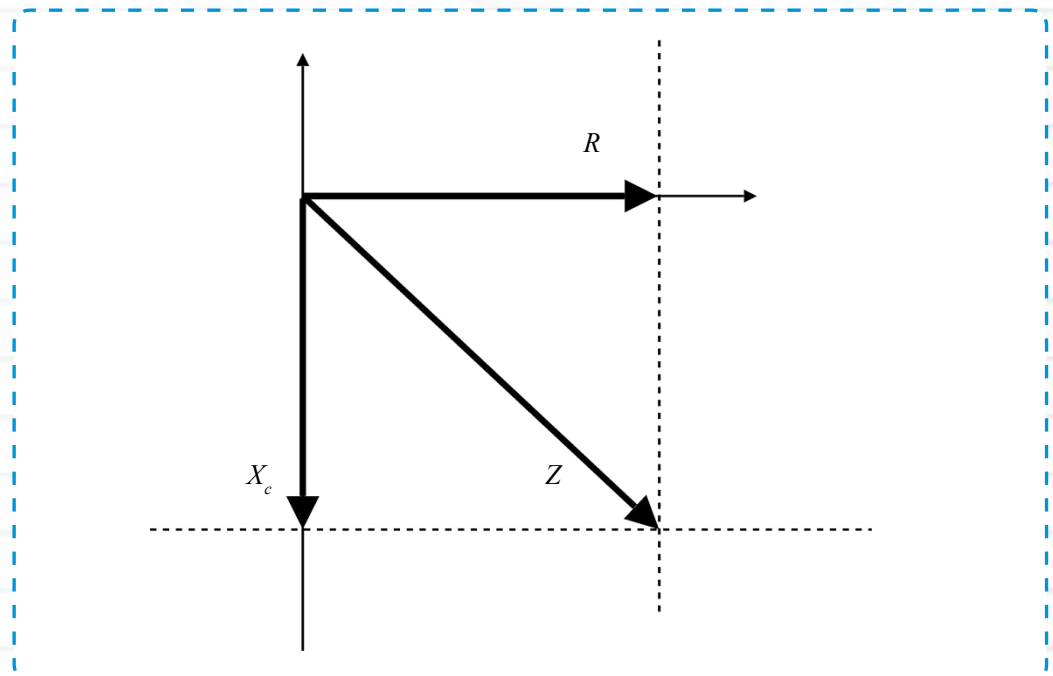
- ✓ Broadband internet data is transmitted on the same line as a voice land-line. To prevent interference between the voice signal and the broadband data signal, an ADSL filter is attached between the telephone outlet and the broadband router.
- ✓ Hearing aids may select a particular range of frequencies associated with speech that have been lost by a person and selectively amplify them. They may also filter out other frequencies not associated with speech to further clarify the sound heard by the user.
- ✓ Active prosthetic limbs that are controlled by current sensors placed on the skin need to respond only to the small deliberate biological signals and must filter out other signal sources such as power lines.

## Activity 2

- 1 When  $C = 1\ \mu\text{F}$  and  $f = 100\ \text{Hz}$ , the reactance of the capacitor is

$$X_c = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 100 \times 1 \times 10^{-6}} = 1.592\ \text{k}\Omega \quad (3\ \text{d.p.})$$

The total impedance of the circuit is given by the vector sum of the resistance and capacitive reactance, see **Figure 6**.



**Figure 6**

Using resistance values in  $\text{k}\Omega$ ,

$$Z = \sqrt{R^2 + X_c^2} = \sqrt{1^2 + 1.592^2} = \sqrt{3.533} = 1.880\ \text{k}\Omega \quad (3\ \text{d.p.})$$



2 The circuit in **Figure 2** is similar to the circuit in **Figure 1**, but with

- $R_1$  replaced by  $R$
- $R_2$  replaced by  $C$

which will replace a resistance of value  $R_2$  by a reactance of  $X_C$ . Similarly, the total resistance of  $R_{tot} = R_1 + R_2$  will be replaced by a total impedance of  $Z = \sqrt{R^2 + X_C^2}$ . The expression for the ratio  $\frac{V_{out}}{V_{in}}$  will be similarly affected:

| Figure 1 circuit                                                       | Figure 2 circuit                                                          |
|------------------------------------------------------------------------|---------------------------------------------------------------------------|
| $\frac{V_{out}}{V_{in}} = \frac{R_2}{R_{tot}} = \frac{R_2}{R_1 + R_2}$ | $\frac{V_{out}}{V_{in}} = \frac{X_C}{Z} = \frac{X_C}{\sqrt{R^2 + X_C^2}}$ |

## Activity 3

1

|                          | Case a                                                                               | Case b                                                                             | Case c                                                                              |
|--------------------------|--------------------------------------------------------------------------------------|------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
|                          | $R = 1 \text{ k}\Omega$ ,<br>$C = 0.1 \text{ }\mu\text{F}$ ,<br>$f = 100 \text{ Hz}$ | $R = 1 \text{ k}\Omega$ ,<br>$C = 1 \text{ }\mu\text{F}$ ,<br>$f = 100 \text{ Hz}$ | $R = 1 \text{ k}\Omega$ ,<br>$C = 10 \text{ }\mu\text{F}$ ,<br>$f = 100 \text{ Hz}$ |
| $X_C$                    | 15.915 k $\Omega$ (3 d.p)                                                            | 1.592 k $\Omega$ (3 d.p)                                                           | 159.155 $\Omega$ (3 d.p)                                                            |
| $Z$                      | 15.947 k $\Omega$ (3 d.p)                                                            | 1.880 k $\Omega$ (3 d.p)                                                           | 1.013 k $\Omega$ (3 d.p)                                                            |
| $\frac{V_{out}}{V_{in}}$ | 0.998                                                                                | 0.847 (3 d.p)                                                                      | 0.157 (3 d.p)                                                                       |

**Table 1**  
Results. Note  
than 0.1  $\mu\text{F}$  =  
100 nF

Using the central results column (*Case b*) as a reference:

- When the value of the capacitor is reduced by a factor 10 (*going from Case b to Case a*), the capacitive reactance increases by a factor of 10. The total impedance does not increase by a factor of 10 as it is calculated as the vector sum of the resistance and reactance. The ratio of  $\frac{V_{out}}{V_{in}}$  increases from 0.847 to 0.998, indicating that the magnitude of the output signal increases.
- When the value of the capacitor is increased by a factor 10 (*going from Case b to Case c*), the capacitive reactance decreases by a factor of 10. The total impedance does not decrease by a factor of 10 as it is calculated as the vector sum of the resistance and reactance. The ratio of  $\frac{V_{out}}{V_{in}}$  decreases from 0.847 to 0.157, indicating that the magnitude of the output signal decreases.

As a general pattern:

- A small value of  $C$  leads to a large capacitive reactance  $X_C$ , which dominates the vector sum used to calculate the total impedance,  $Z$ , for the  $R$  value used. In this case the magnitude of the impedance is similar to the magnitude of the capacitive reactance.
- A large value of  $C$  leads to a small capacitive reactance  $X_C$ , which is dominated in the vector sum used to calculate the total impedance,  $Z$ , by the resistor value,  $R$ . In this case the magnitude of the impedance is similar to the magnitude of the resistance.

2

|                          | Case a                                                                            | Case b                                                                             | Case c                                                                            |
|--------------------------|-----------------------------------------------------------------------------------|------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------|
|                          | $R = 1 \text{ k}\Omega$ ,<br>$C = 1 \text{ }\mu\text{F}$ ,<br>$f = 10 \text{ Hz}$ | $R = 1 \text{ k}\Omega$ ,<br>$C = 1 \text{ }\mu\text{F}$ ,<br>$f = 100 \text{ Hz}$ | $R = 1 \text{ k}\Omega$ ,<br>$C = 1 \text{ }\mu\text{F}$ ,<br>$f = 1 \text{ kHz}$ |
| $X_c$                    | 15.915 k $\Omega$ (3 d.p)                                                         | 1.592 k $\Omega$ (3 d.p)                                                           | 159.155 $\Omega$ (3 d.p)                                                          |
| $Z$                      | 15.947 k $\Omega$ (3 d.p)                                                         | 1.880 k $\Omega$ (3 d.p)                                                           | 1.013 k $\Omega$ (3 d.p)                                                          |
| $\frac{V_{out}}{V_{in}}$ | 0.998                                                                             | 0.847 (3 d.p)                                                                      | 0.157 (3 d.p)                                                                     |

**Table 2**  
**Results**

Using the central results column (*Case b*) as a reference:

- When the value of the frequency is reduced by a factor 10 (*going from Case b to Case a*), the capacitive reactance increases by a factor of 10. The total impedance does not increase by a factor of 10 as it is calculated as the vector sum of the resistance and reactance. The ratio of  $\frac{V_{out}}{V_{in}}$  increases from 0.847 to 0.998, indicating that the magnitude of the output signal increases. Note, this gives the same result as keeping the frequency at 100 Hz and reducing the Capacitance by a factor of 10, which is not surprising when you consider the expression for capacitive reactance is  $X_c = \frac{1}{2\pi fC}$ , i.e. the effects of  $f$  and  $C$  are multiplied in the denominator.
- When the value of the frequency is increased by a factor 10 (*going from Case b to Case c*), the capacitive reactance decreases by a factor of 10. The total impedance does not decrease by a factor of 10 as it is calculated as the vector sum of the resistance and reactance. The ratio of  $\frac{V_{out}}{V_{in}}$  decreases from 0.847 to 0.157, indicating that the magnitude of the output signal decreases.

As a general pattern:

- A small value of  $f$  leads to a large capacitive reactance  $X_c$ , which dominates the vector sum used to calculate the total impedance,  $Z$ , for the  $R$  value used. In this case the magnitude of the impedance is similar to the magnitude of the capacitive reactance.
- A large value of  $f$  leads to a small capacitive reactance  $X_c$ , which is dominated in the vector sum used to calculate the total impedance,  $Z$ , by the resistor value,  $R$ . In this case the magnitude of the impedance is similar to the magnitude of the resistance.

3

From the results above it can be seen that for small values of  $f$ ,  $X_c$  will be large and  $\frac{V_{out}}{V_{in}}$

will be close to 1. For large values of  $f$ ,  $X_c$  will be small and  $\frac{V_{out}}{V_{in}}$  will be close to zero. This

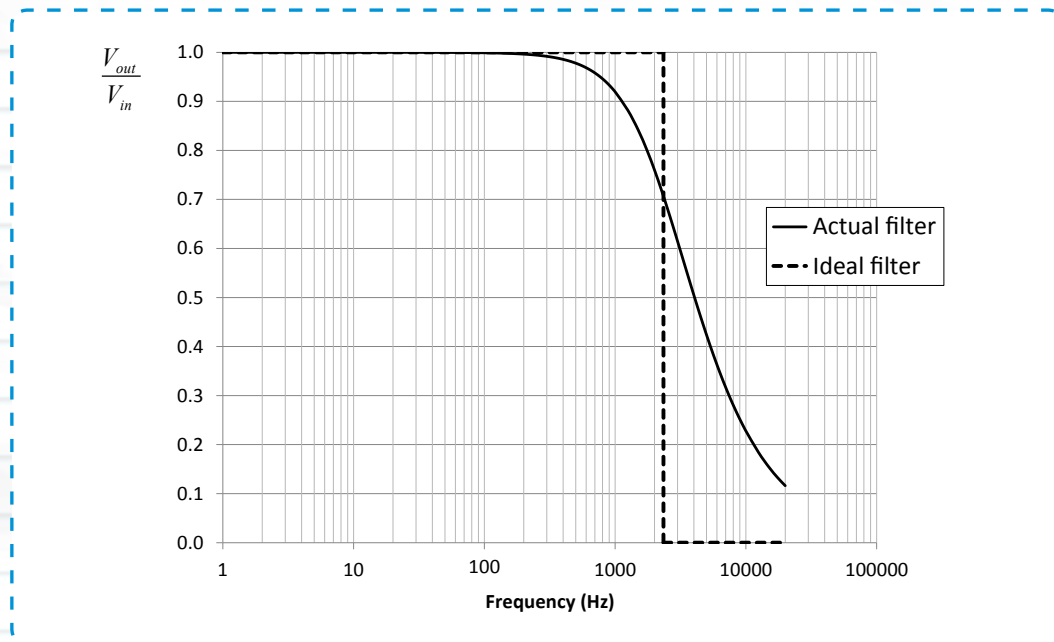
behaviour gives a simple filter – low frequencies will give an output that is similar in magnitude to the input, i.e. they will be passed. High frequencies will give an output that is close to zero, i.e. they will be filtered out. Because of this behaviour the circuit is called a ‘low pass filter’.

4

$R = 680 \text{ }\Omega$ ,  $C = 100 \text{ nF}$  gives a cut-off frequency of 2.341 kHz. However, this is not a unique pair, for example  $R = 1 \text{ k}\Omega$  and  $C = 68 \text{ nF}$  gives the same cut-off frequency. There are other such pairs.

5

The filter behaviour isn’t ideal. An ideal filter would pass all frequencies below 2.341 kHz and block all frequencies above it, so the frequency response would be a ‘step function’. The resource shows that instead of behaving like this, the circuit gradually reduces how much of the signal above the cut-off frequency is passed. See **Figure 7**.



**Figure 7**  
Ideal and actual  
filter behaviour

## Activity 4

- 1 For a given set of  $R$  or  $C$  values the cut-off frequency for a high pass filter is the same as the cut-off frequency for a low pass filter.
- 2 As with the low pass filter, increasing  $R$  or  $C$  reduces the cut-off frequency. Reducing  $R$  or  $C$  increases the cut-off frequency.
- 3  $R = 4.7 \text{ k}\Omega$ ,  $C = 680 \text{ nF}$  gives a cut-off frequency of 49.798 Hz. However, this is not a unique pair, for example  $R = 6.8 \text{ k}\Omega$  and  $C = 470 \text{ nF}$  gives the same cut-off frequency. There are other such pairs.
- 4 This circuit still has the same impedance,  $Z = \sqrt{R^2 + X_c^2}$ . However, the output is given as the voltage drop across the resistor rather than the capacitor so that

$$\frac{V_{out}}{V_{in}} = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + X_c^2}}.$$

For small values of  $f$ ,  $X_c$  will be large and  $\frac{V_{out}}{V_{in}}$  will be close to 0, i.e.  $V_{out}$  will be small. For

large values of  $f$ ,  $X_c$  will be small and  $\frac{V_{out}}{V_{in}}$  will be close to 1, i.e.  $V_{out}$  will be close to  $V_{in}$ .





## Stretch and Challenge

- 1 A low pass filter has a negative phase shift and the angle of shift becomes more negative as the magnitude of signal that is passed decreases, i.e. the shift is small for frequencies below the cut-off frequency and becomes more negative for frequencies higher than the cut-off frequency. The shift is never less than  $-90^\circ$ . This is shown on the plot as the red trace for  $V_{out}$  lagging behind the blue trace for  $V_{in}$ . The cut-off frequency coincides with a phase shift of  $-45^\circ$ .

A high pass filter has a positive phase shift and the angle of shift becomes larger as the magnitude of signal that is passed decreases, i.e. the shift is small for frequencies above the cut-off frequency and increases for frequencies lower than the cut-off frequency. The shift is never more than  $90^\circ$ . This is shown on the plot as the red trace for  $V_{out}$  leading the blue trace for  $V_{in}$ . The cut-off frequency coincides with a phase shift of  $45^\circ$ .

- 2 and 3 Calculator exercise.

- 4 For the low pass filter  $\frac{V_{out}}{V_{in}} = \frac{X_c}{Z} = \frac{X_c}{\sqrt{R^2 + X_c^2}}$ . The phase angle for  $X_c$  is  $-90^\circ$ . The phase angle for  $Z$  is given by  $\tan^{-1}\left(\frac{X_c}{R}\right)$ . Using the values from the image

$$R = 4.7 \text{ k}\Omega$$

$$X_c = 4.823 \text{ k}\Omega$$

$$Z = 6.734 \text{ k}\Omega$$

The phase angle for  $Z$  is  $\tan^{-1}\left(\frac{-4.823 \times 10^3}{4.7 \times 10^3}\right) = -45.7$ . Note, the negative sign of  $X_c$  in this expression; in the phasor diagram, capacitive reactance points down.

The phasor division is now

$$\frac{V_{out}}{V_{in}} = \frac{X_c \angle -90^\circ}{Z \angle -45.7^\circ} = \frac{4.823 \times 10^3 \angle -90^\circ}{6.734 \times 10^3 \angle -45.7^\circ} = \frac{4.823 \times 10^3}{6.734 \times 10^3} \angle -90^\circ - (-45.7^\circ) = 0.716 \angle -44.3^\circ$$

Which gives the correct value of  $\frac{V_{out}}{V_{in}}$  and the correct phase shift angle.

For the high pass filter  $\frac{V_{out}}{V_{in}} = \frac{R}{Z}$ . The phasor division for this is

$$\frac{V_{out}}{V_{in}} = \frac{R \angle 0^\circ}{Z \angle -45.7^\circ} = \frac{4.7 \times 10^3 \angle 0^\circ}{6.734 \times 10^3 \angle -45.7^\circ} = \frac{4.7 \times 10^3}{6.734 \times 10^3} \angle 0^\circ - (-45.7^\circ) = 0.698 \angle 45.7^\circ$$

Again giving the correct value of  $\frac{V_{out}}{V_{in}}$  and the correct phase shift angle.

